Institutional Crowding and the Moments of Momentum

Pedro Barroso, Roger M. Edelen, and Paul Karehnke*

October 1, 2017

Abstract

Several studies outline a destabilizing effect on asset prices from arbitrageur crowding. We consider this as an explanation for momentum crashes. We first develop theoretical predictions under a variety of belief mechanisms, and then test those predictions using 13F institutional holdings data. Our theory predicts unbounded momentum crashes when arbitrageurs maintain naïve linear beliefs, but no abnormal tail risk when beliefs are a fixed point of market clearing. Consistent with the equilibrium predicted under rational beliefs, our empirical tests on the first four moments of momentum returns find no evidence that momentum crashes relate to institutional crowding.

Keywords: Momentum, crash risk, institutional investors, crowded trade, destabilize

*Barroso and Karehnke are at the University of New South Wales, Edelen is at UC Davis. Emails: p.barroso@unsw.edu.au, rmedelen@ucdavis.edu, p.karehnke@unsw.edu.au. We thank seminar participants at the University of Arizona, New South Wales, California Riverside, Technology at Sydney, Newcastle, Virginia Tech, Tennessee's Smokey Mountain Finance Conference, The 14th Annual IDC/Herzliya Conference and Annual Quantitative Trading Symposium, and Manchester's Hedge Fund Conference for their helpful comments and suggestions. We are especially grateful for comments from John Griffin, Hening Liu, Rick Sias, and Neal Stoughton. Semyon Malamud was particularly helpful to the paper's development. Errors are the authors' responsibility.

1 Introduction

Several studies argue that arbitrageur capital can lead to destabilized prices as a result of incomplete information regarding the market setting. For example, Abreu and Brunnermeier (2003) show that if arbitrageurs cannot coordinate an attack, financial bubbles can be sustained by arbitrageurs themselves, who trade against fundamental value to 'ride' the bubble. Stein (2009) shows that uncertainty regarding the crowding of other arbitrageurs can similarly lead to overstated beliefs that push prices away from fundamental value. In the specific context of momentum, which is the focus of this study, Lou and Polk (2013) and Huang (2015) both argue that crowding by momentum investors potentially explains negative skewness in momentum returns. Edelen et al. (2016) relate institutional purchases to a broad array of anomaly returns, including momentum, and document a crowding effect on the level of returns. Thus, there is both theoretical and empirical support for the hypothesis that crowding by momentum investors generates momentum crashes.

We directly investigate this hypothesis from a theoretical and empirical perspective. We first develop a model similar in spirit to Stein (2009) to demonstrate how different belief mechanisms employed by momentum investors lead to different predictions regarding the impact of crowding on momentum returns. In addition to the linear setting considered in his analysis, we solve for fixed-point beliefs in the market-clearing equation. Though nonlinear and rather complicated to solve, such beliefs are required of rational momentum investing. We demonstrate the important role of beliefs with simulations that highlight the consistency of fixed-point beliefs and the inconsistency and inefficiency of even optimized linear beliefs.

More importantly, with a fixed-point characterization of arbitrageur beliefs momentum tail risk does not differ from the primitive distribution of underlying asset returns. There are no momentum crashes. Conversely, when arbitrageurs ignore crowding uncertainty in an otherwise identical setting, momentum tail risk is enormous, with unbounded crashes and negative skewness and excess kurtosis in strategy returns many orders of magnitude beyond that of primitive returns. Thus, our analysis emphasizes that crowding-induced momentum crashes derive from irrationality in arbitrageur beliefs, not crowding per se. Likewise, empirical tests relating proxies for arbitrageur crowding to higher moments of momentum returns are best interpreted as tests of the rationality of arbitrageur beliefs regarding tail risk, rather than tests of a crowding effect per se.

Intuitively, the key aspect of the fixed-point solution is that momentum investors directly compute the conditional probability distribution for fundamental value given a price signal (i.e., differential return on winners and losers during the portfolio formation period). By definition of a fixed-point, the demands that momentum investors formulate using this probability distribution lead to an equilibrium price that confirms the distribution. Thus, rational momentum investors are fully aware of the potentially destabilizing effects of unanticipated crowding. But since they optimize over the correct probability distribution for returns, including any equilibrium tail risk that arises from crowding, equilibrium tail risk must be insignificant. Otherwise, the devastating impact that crash probability has on expected utility would cause momentum investors to alter their demands, changing the probability distribution for returns and invalidating the proposed equilibrium.

We demonstrate this with simulations under each belief mechanism. Ex ante (log) momentum returns[1] have the following characteristics for naïve versus fixed-point beliefs, respectively:

- Mean of -2.4% versus 3.0%, and a minimum of $-39,000\%$ versus -2.6%,
- Standard deviation of 174% versus 1.6%,
- Skewness of -151.3 versus 0.4, and kurtosis of $30,000$ versus 3.0,
- Certainty equivalent return of *complete loss* versus 2.5%.

Thus, our theory predicts that crowding can generate virtually unbounded ex ante tail risk if arbitrageurs maintain naïve beliefs, but that crowding has no impact on tail risk in an otherwise identical setting with fixed-point beliefs. Unanticipated crowding predicts crashes if and only if arbitrageurs are naïve in their beliefs.

Our empirical analysis uses 13F institutional holdings data to construct proxies for momentum investing by directly examining how holdings correlate with momentum prescriptions (i.e.,

[1] That is, the expected return conditioning on all information available as of the portfolio formation date.

Jegadeesh and Titman, 1993). Our proxies consider both anticipated and unanticipated measures of momentum demands, and they decompose those demands into crowding by arbitrageur peers (i.e., number of momentum investing institutions) and the trading intensity of a representative arbitrageur (i.e., capital allocations of momentum investing institutions). To our knowledge, this direct and comprehensive construction of proxies for institutional momentum investing is novel and provides an incremental contribution beyond testing the models predictions regarding pricing effects from crowding.

We find strong evidence that crowding negatively predicts mean momentum returns. This is consistent with our theory under all belief specifications. However, we find little evidence that crowding positively predicts tail risk. To be meaningful, tail risk implies negative skewness, elevated volatility, and excess kurtosis; not just negative skewness. Our proxies for institutional crowding in the momentum strategy *negatively* relate to all three higher moments, often with statistical reliability. This surprising result is not consistent with a causal role for crowding in momentum crashes, but it is consistent with rational momentum investors optimizing their demands to account for time-varying toxicity in market conditions. We provide a broad range of supporting evidence for this assessment of arbitrageur beliefs, based on nuanced differences in the proxies. We conclude that the evidence supports our theoretical analysis using rational, fixed-point beliefs.

Overall, our analysis rejects the hypothesis that crowding plays a causal role in momentum crashes—on both theoretical and empirical grounds. Rather, we find strong support for the alternative hypothesis that arbitrageurs employ a rational belief-formation mechanism that identifies and attenuates—indeed, eliminates—the potentially destabilizing effects of crowding uncertainty.

There is much related literature on the subject. We provide a detailed survey in Appendix A. Section 2 develops the model and Section 3 develops and analyzes its result using a simulation approach. Section 4 presents the empirical analyses and Section 5 concludes the study.

2 Model

Section 2.1 lays out the assumptions and setting of the model and Section 2.2 develops four solutions to the equilibrium, differing by how momentum traders form their beliefs.

2.1 Setting

To save space we work directly with the momentum portfolio, which can be developed from individual stocks as in Appendix B. The key components in that development are as follows. The formation period begins with all investors holding the market portfolio but some (informed investors) observe a common signal δ of differential fundamental value $d = \delta + \epsilon$ for a subset of stocks (positive for winners, negative for losers). The trading of informed investors in the formation period identifies the momentum portfolio by way of a price signal. We employ a two period setting with all trading occurring in a call auction at the end of the first period (portfolio formation date) and payoffs realized at the end of the second (evaluation) period.

Let f denote the formation-period log return on winners minus loser stocks. We generally refer to f as the price of the momentum portfolio. The realized momentum return is

$$m + \epsilon = d - f. \tag{1}$$

We refer to m as the ex ante momentum return (= $\delta - f$, the expected post-formation date return on the momentum portfolio conditional on all information in the economy as of the portfolio formation date). Most of our analysis pertains to m rather than realized momentum returns.

The price f of the momentum portfolio is determined by balancing the demands of three investor groups. Informed investors with capital K_I observe and directly respond to δ, with

$$E_I\left[m + \epsilon \,|\delta, f\right] = \delta - f, \qquad Var_I\left[m + \epsilon \,|\delta, f\right] = \sigma_d^2. \tag{2}$$

Thus, m is informed investors' expectation of momentum returns. Momentum investors with capital K_M do not observe m, but they infer it from the price, f. Let $\delta^E = E_M[\delta|f]$ and $\delta^V = Var_M[\delta|f]$ denote their expectations (further discussed in Section 2.2)

$$E_M[m+\epsilon|f] = \delta^E - f, \qquad Var_M[m+\epsilon|f] = \delta^V + \sigma_d^2. \tag{3}$$

Counterparty investors with capital K_C trade counter to the price moves caused by informed and momentum investor demands, fixating on historical information with $E_C d = 0$:

$$E_C[m+\epsilon|f] = -f, \qquad Var_C[m+\epsilon|f] = \sigma_\delta^2 + \sigma_d^2. \tag{4}$$

All investors hold power utility preferences with relative risk aversion γ, maximizing[2]

$$\log E[u(K)] = \log E\left[\frac{K^{1-\gamma}}{1-\gamma}\right] \tag{5}$$

on the portfolio-formation date, where K generically references end of period wealth. We use the methodology of Campbell and Viceira (2002, appendix) to derive the following expression

$$Demand = \frac{E_{type}[m+\epsilon]}{\gamma Var_{type}[m+\epsilon]} K_{type}, \tag{6}$$

where K_{type} is the beginning of period capital of a given 'type' investors, and details are in Appendix C. Applying each of the three investor types' expectations and capital allocations to (6), summing, and equating to zero supply gives the market clearing condition

$$f = \frac{1}{D}\left(\delta k_I + \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right), \tag{7}$$

where $D = \left(1 - \frac{\delta^V}{\sigma_d^2+\delta^V} k_M\right)$ and $k_{type} = K_{type} / \left(\frac{\sigma_d^2}{\sigma_d^2+\sigma_\delta^2} K_C + K_I + K_M\right)$ indicates fraction of capital, with counterparty capital adjusted for dividend variance relative to informed investors.

[2] We assume that $\gamma > 1$ without loss of generality throughout the paper.

2.2 Solutions

We consider four solutions to Eq. (7), differing by how momentum investors' form beliefs. In the base case capital allocations are known and momentum investors correctly formulate linear beliefs. In the second, capital allocations are stochastic but momentum investors ignore this fact and treat them as fixed at their expected values, again applying linear beliefs. In the third, momentum investors maintain linearity in their beliefs but choose those beliefs optimally to account for noise and potential tail risk from capital uncertainty. In the fourth, momentum investors use the distributions for δ, k_M, and k_I to construct a probability distribution for dividends that correctly assimilates equilibrium crowding effects.

2.2.1 Linear beliefs with known capital

First consider the case of known capital k_M and k_I. If momentum investors conjecture a linear equilibrium

$$f = \lambda\delta \tag{8}$$

with $\lambda \equiv k_I + k_M$, then beliefs $\delta^E = \lambda^{-1} f$ and $\delta^V = 0$ lead to a self-fulfilling, linear solution to Eq. (7).[3] That is, Eq. (8) gives the resulting dependence of prices on δ, and $\lambda^{-1} f$ and 0 are the conditional expectation and variance, respectively.

2.2.2 Naïve linear beliefs (capital uncertainty that momentum investors ignore)

With uncertain capital allocations the linear conjecture in Section 2.2.1 for the market-clearing pricing relation is problematic. For example, Stein (2009) uses this conjecture of a linear pricing relation in the context of news events and concludes that uncertain crowding by arbitrage capital can cause arbitrageurs to destabilize prices rather than drive prices towards efficiency. Therefore we now assume that both informed and momentum capital proportion are stochastic, but momentum investors treat both as constants equal to their expected values. Hence they form beliefs using

[3] Note that Eq. (8) implies that δ is known given f.

$$f = \lambda_E \delta, \qquad \text{where} \qquad \lambda_E \equiv Ek_M + Ek_I. \tag{9}$$

We refer to this belief mechanism as 'naïve beliefs,' under which the market clears at the price

$$f = \quad \lambda_E \left(\frac{k_I}{\lambda_E - k_M} \right) \delta \quad = \quad \lambda_E \left(\frac{k_I}{Ek_I - (k_M - Ek_M)} \right) \delta. \tag{10}$$

Eq. (10) identifies the problem with failing to account for capital uncertainty in forming momentum demands. While the bracketed multiplier in Eq. (10) equals one when capital allocations happen to equal their expected values, that term generally contributes a stochastic element to the *realized* coefficient on δ in Eq. (9). This has important implications for the moments of the distribution of momentum returns, and for the inference problem that momentum investors face.

First, consider the distribution of momentum returns under naïve beliefs. As unanticipated momentum capital $k_M - Ek_M$ approaches Ek_I, the bracketed multiplier in Eq. (10) grows without bound. Consider the extreme cases of $(k_M - Ek_M) \geq Ek_I$, or $k_M \geq \lambda_E$. Recalling that $\delta^E = \lambda_E^{-1} f$ and $\delta^V = 0$ under naïve beliefs, we can write the market clearing Eq. (7) as

$$f(1 - k_M) - \delta k_I = f(1 - \lambda_E) \frac{k_M}{\lambda_E}, \tag{11}$$

where the left-hand side is the supply of the momentum portfolio to momentum investors and the right-hand side is their demand for it. When $k_M = \lambda_E$ momentum investors' demand exceeds supply at all values of f, implying unrelenting upward pressure on the price of the momentum portfolio. As a result, its valuation on the portfolio formation date can exceed ex ante fundamental value (δ) by a potentially unbounded margin, implying virtually infinite negative ex ante momentum returns. The case of $k_M > \lambda_E$ is similar, except that here momentum investors' demands are unrelenting *and increasing* in f. Thus, the same argument applies.[4]

The nonlinearity of this potential price destabilization implies kurtosis in momentum returns.

[4]There is, however, a equilibrium at a finite negative value for f. We consider this case in Appendix D but note here that it too predicts extreme negative ex ante momentum returns.

But note that this kurtosis resides primarily on the left tail. When $k_M - Ek_M$ takes on extreme low, rather than high, values (with the minimum occurring at $k_M = 0$), the market mainly (or entirely) consists of informed investors trading against counterparty investors. Such a market is well-behaved, clearing at $f = \delta k_I$. Combining, we conclude that under naïve beliefs unanticipated momentum capital induces negative skew as well as excess kurtosis in momentum returns.

The second implication of the stochastic coefficient in Eq. (10) is problematic inferences of δ from f. The deviation between the actual coefficient in the dependence of f on δ (from Eq. (10)) and that which momentum investors presume (from Eq. (9)) implies noise in inferences, similar to an errors-in-variables problem. A more consistent slope would presume a higher value of λ_E; i.e., lower value of λ_E^{-1} to account for the loss of signal. We summarize the remarks in this section as

Result 1 *When momentum investors maintain naïve beliefs as expressed in Eq.* (9) *the equilibrium probability distribution of ex ante momentum returns contains arbitrarily large tail risk in the form of high variance, negative skewness, and excess kurtosis. Conditional values of these higher moments are increasing in the ex ante variance of unanticipated momentum capital, $\sigma^2_{k_M}$, and in the ex post realization of unanticipated momentum capital, $k_M - Ek_M$. The mean of ex ante momentum returns is decreasing in $\sigma^2_{k_M}$ and $k_M - Ek_M$.*

2.2.3 Optimal linear beliefs

One way that momentum investors might alter their beliefs so as to avoid – or at least mitigate – the destabilizing effects of capital uncertainty is to lessen the slope coefficient λ^{-1} in $\delta^E = \lambda^{-1} f$. This is analogous to an errors-in-variables adjustment in a regression context, and has the effect of lessening the stochastic multiplier in Eq. (10). Let us denote the adjusted linear coefficient that maximizes expected utility as λ_*^{-1} and refer to the corresponding beliefs as optimal linear beliefs.

Optimal linear beliefs are generally not optimal overall. A reduction in the slope of beliefs while maintaining linearity in δ^E means inhibiting momentum trading equally on the margin. Yet, the conditional expectation of crowding-induced overvaluation in f is increasing in f. Thus, restriction to a linear solution implies an optimum with too much crash risk at high values of f and too much sacrificing of momentum profits at moderate to low values of f. A nonlinear solution, as developed in Section 2.2.4, can optimize locally, improving overall efficiency. Finally, note that unless the investor maintains a belief coefficient $\lambda_* = 1$ or the distribution of k_M can be bounded away from 1 on prior grounds, there is always some possibility of an unbounded, crowding-induced crash (a zero in the denominator of Eq. (10)). Generally, to eliminate crowding-induced crashes momentum investors must hold beliefs $\lambda_* = 1$, which precludes momentum trading altogether.

We summarize this as

Result 2 *Optimal linear demands (where beliefs of ex ante fundamental value are restricted to be a linear function of price) generally result in more tail risk and offer less certainty equivalence in momentum profits than a nonlinear belief mechanism. In particular, optimal linear demands always retain some probability of an unbounded crash unless restrictions are placed on the distribution of capital allocations.*

2.2.4 Direct calculation of beliefs as a fixed-point solution to Eq. (7)

The nonlinear optimum solves for the fixed-point of momentum investors' beliefs at each value of f in the market clearing Eq. (7). The random variables in the system are δ, k_I, and k_M. We write their joint density as $g(\delta)\, h(k_M, k_I)$, noting that δ is independent of k_M and k_I but that the k's themselves are clearly dependent. Momentum investors seek to compute δ^E and δ^V given this distribution and observation of f:

$$\begin{aligned} \delta^E &= \int_0^\infty \delta p(\delta|f)\, d\delta = \int_0^\infty \delta \frac{p_2(\delta, f)}{p_1(f)} d\delta, \\ \delta^V &= \int_0^\infty \left(\delta - \delta^E\right)^2 \frac{p_2(\delta, f)}{p_1(f)} d\delta, \end{aligned} \tag{12}$$

where numerical subscripts serve to distinguish the probability density functions. To solve for these densities we transform the primitive random variable k_I into the observable random variable

f using Eq. (7). Under this transformation the market clears at an f with pdf

$$p_3(\delta, k_M, f) = g(\delta)\, h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right)\right)\frac{D}{\delta}, \tag{13}$$

(see Appendix E). Integrating Eq. (13) gives the pdfs we seek

$$\begin{aligned} p_2(\delta, f) =& \frac{g(\delta)}{\delta}\int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right)\right) D dk_M, \\ p_1(f) =& \int_0^\infty \frac{g(\delta)}{\delta}\int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right)\right) D dk_M d\delta. \end{aligned} \tag{14}$$

The fixed-point expressions for beliefs are

$$\delta^E = \frac{\int_0^\infty g(\delta)\int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right)\right) D dk_M d\delta}{\int_0^\infty \delta^{-1} g(\delta)\int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right)\right) D dk_M d\delta}, \tag{15a}$$

and

$$\delta^V = \frac{\int_0^\infty \frac{\left(\delta-\delta^E\right)^2}{\delta} g(\delta)\int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right)\right) D dk_M d\delta}{\int_0^\infty \delta^{-1} g(\delta)\int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right)\right) D dk_M d\delta}. \tag{15b}$$

Unlike the previous three specifications where beliefs are linear in f, we know of no closed-form solution for fixed-point beliefs. The approach we take is to solve for δ^E and δ^V conditioning on a given value of f,[5] repeating on a fine grid over the plausible range of values for f. This yields a discrete approximation to the continuous belief mappings $f \to \delta^E$ and $f \to \delta^V$. We then approximate the mapping at arbitrary values of f by interpolation.

[5]In particular, we use Matlab's FSolve function to jointly locate the roots of the LHS minus RHS for Eqs. (15a) and (15b) for a given f.

3 Simulations

In this section we analyze the equilibrium under each of the four specifications of beliefs.

3.1 Methodology and assumptions

The approach is to randomly generate 100,000 draws of the primitive random variables δ, k_M and k_I and solve for the market clearing f under each draw using Eq. (7) and the indicated specification of beliefs. We then use those 100,000 simulation trials to evaluate the consistency of beliefs and the behavior of realized momentum returns.

We assume that k_I and k_M follow a Dirichlet distribution, Dir($[\alpha_1 \ \alpha_2 \ \alpha_3]$), which provides a natural dependence between the two as fractions of a whole, with a marginal distribution for each that follows the Beta distribution. We parametrize the Dirichlet to yield equal expected capital allocations of 1/3, with all concentration parameters α_i equal to 12. We explore the impact of variance in k_M on market equilibrium by varying the concentration parameters.

We use a lognormal distribution for δ with parameters $\mu = -2.405$ and $\sigma = 0.125$.[6] We also consider a uniform distribution in the Internet Appendix. Finally, ϵ (used to compute the realized momentum returns as in Eq. (1)) is drawn from a zero-mean normal distribution with $\sigma_d = 0.125$. While most of our results are based on the ex ante momentum return, realized momentum returns are used to calculate certainty equivalence and to choose optimal linear beliefs. Momentum investors know all probability distributions.

We use three plots to analyze results in each of the four belief specifications, as shown in Figure 1. To construct each plot, the 100,000 simulation trials under each belief specification are ranked into 100 bins according to the horizontal-axis variable. Averages are then computed within each bin to approximate the conditional expectation of the variable indicated on the vertical axis, and the value of the conditioning variable on the horizontal axis.

[6]These values imply an average δ of 9.1% with standard deviation of 1.14%. The log-normal distribution has the advantage that the differential dividend is limited to positive values, and this distributional assumption is consistent with the evidence in Andersen et al. (2001).

The first row of plots depicts the consistency of beliefs, with trials ranked according to beliefs δ^E and the average δ underlying those beliefs (i.e., the average draw of δ in the simulation trials that find their way into the corresponding bin) plotted on the vertical axis. Rational beliefs imply an identity mapping. The second row of plots indicate tail risk, with the ex ante momentum return on the vertical axis and beliefs on the horizontal axis. The third row depicts the impact of unanticipated momentum capital on market equilibrium, with the same vertical axis as the second plot. Finally, we consider two parameterizations of $\sigma^2_{k_M}$ in each plot.

Table 1 reports on descriptive statistics of unconditional expected momentum returns from these simulations, including the certainty equivalent return for a momentum investor with $\gamma = 2, 4, 10$ and the expected dollar return using $\gamma = 2$. We label the latter 'profit' and note that a value of 1 (-1) means that investors' capital increases (decreases) at an expected rate of 100% (-100%) per momentum cycle. (Recall that we use log returns (except for certainty equivalent returns) so the rate of increase or decline can be arbitrarily large.)

3.2 Simulation results

We discuss the results depicted in Figure 1 and Table 1 under each of the four belief specifications.

[Insert Figure 1 and Table 1 near here]

Known capital allocations. In the simplest specification k_M and k_I are random but directly observed by momentum investors each trial, prior to trading. The values derived in this base case provide a useful benchmark for considering the more realistic remaining cases.

Since the market clearing price is here a deterministic function of δ, momentum investors effectively observe δ via f. Thus Plot A.1 of Figure 1 depicts an exact identity. Plot A.2 depicts the ex ante momentum return m against beliefs, yielding a line with slope equal to the average value of $1 - k_M - k_I = 1/3$. Not surprisingly, there are no crashes in this setting, as evidenced by the low standard deviation, small positive skewness, and low excess kurtosis in Table 1, Panel A.

Finally, from Plot A.3, m is negatively related to unanticipated momentum capital, consistent with the intuition that higher demand leads to a higher-formation period return and a lower subsequent momentum return.

Naïve beliefs. Panel B of Figure 1 and Table 1 present the case of unobserved k_M and k_I with momentum investors holding linear beliefs with $\lambda_E = 2/3$. The beliefs Plot B.1 indicates a concave relation between δ^E and δ: When momentum investors do not account for capital uncertainty their beliefs increasingly overstate fundamental value at higher values of f. This is particularly so when $\sigma^2_{k_M}$ is high. For example, in the top bin of δ^E (highlighted with $\rightarrow$) beliefs are about 5.95 while the actual δ supporting the f that momentum investors are conditioning upon averages just 0.007.[7]

Not surprisingly, this gross overstatement of beliefs leads to a substantially negative ex ante momentum return—i.e., a predictable momentum crash. Thus, while the ex ante momentum return in the extreme bin of Plot A.2 (case of known capital) is about 4%, it is -393% (log return) in the high $\sigma^2_{k_M}$ case under naïve beliefs. The unconditional distribution of expected momentum returns in Panel B of Table 1 likewise exhibits substantial volatility, extreme negative skewness and excess kurtosis under naïve beliefs. The certainty equivalent utility implies that total loss is assured.[8] Plot B.3 from Figure 1 shows the source of these catastrophic returns: the ex ante momentum returns with the second highest bin of $k_M - Ek_M$ mirrors that seen in the worst bin of Plot B.2. Notice that crashes in the highest $k_M - Ek_M$ bin are smaller than in the second highest bin because momentum returns there are determined by the $-f$ equilibrium described in Appendix D.

Panel B of Figure 1 and Table 1 make it clear that uncertain competition in the momentum strategy can indeed generate predictable momentum crashes. Panel A shows that this is not the

[7] Notice that the average δ in the highest bin is very small because this bin included the negative f equilibria, in which momentum investors buy stocks with $\iota = -1$ and short stocks with $\iota = 1$ and effectively face $-\delta$ on their long short portfolio. See also Footnote 4 and Appendix D.

[8] Notice that the certainty equivalent return of -1 are obtained due to our use of approximate solutions for optimal demands which effectively ignore crash risk. Faced with these prospects, investors would presumably decide not to buy the momentum portfolio and earn a certainty equivalent return of 0 instead. Closed form solutions to such a problem do not exist, however, making it necessary to rely on the approximate solutions. This approach is in the spirit of Stein (2009), who assumes demands of a similar functional form than ours. In addition, these approximate solutions appear to be reliable in the known capital allocations, optimal linear beliefs, and fixed-point beliefs case.

case when momentum investors can condition on k_M, irrespective of its variance $\sigma^2_{k_M}$. Thus, as Stein (2009) notes, it is not variance in arbitrage capital that causes destabilization but rather unobservability in that variance. This conclusion of predictable crashes, however, presumes naïve linear beliefs. We now consider what happens when momentum investors optimize those beliefs.

Optimal linear beliefs. In constructing optimal linear beliefs we do not attempt to directly formulate λ_*. Rather, we simulate momentum returns and demands as a function of a proposed λ ranging from the known-capital case of $2/3$ up to 1. We then select the value that yields the highest average realized utility (Eq. (5)) over the 100,000 simulation trials drawn for each proposed λ. In this exercise we use a coefficient of relative risk aversion γ equal to 2, though we also consider $\gamma = 4$ and 10 in untabulated analyses (results qualitatively identical). Table 1 presents λ_*^{-1}, equal to 1.38 (1.12) in the low (high) $\sigma^2_{k_M}$ case.

Plot C.1 of Figure 1 depicts the performance of beliefs under this optimal linear specification. The first-order impact of this optimization is elimination of the run-away overstatement of beliefs that occurs when arbitrageurs ignore crowding effects (i.e., that seen in the extreme bins of Plot B.1). Here, the underlying true δ corresponding to investors' beliefs $\delta^E = E(\delta|f)$ are at least monotonically increasing in those beliefs. However, the slope of the plots are nowhere near one, as required under rationality. Investors still overstate fundamental value when beliefs are high (fueling tail risk), and they now understate value when beliefs are low—particularly in the case of high $\sigma^2_{k_M}$ where arbitrageurs are most keen on avoiding tail risk. Linearity forces a coupling of the two poles of beliefs, a condition not imposed with fixed-point beliefs (Panel D), where rational investors get the conditional expectation of δ right at all values of the conditioning variable, f.

Fixed-point beliefs. The inefficiency of optimal linear beliefs relative to rational beliefs is evident in Table 1. For example, comparing the "high" columns of Panels C and D, optimal linear beliefs lead to negative skewness with a minimum ex ante momentum return of −53% versus positive skewness with a minimum ex ante momentum return of −2.6% for fixed-point beliefs. Fixed-point beliefs are five times more profitable with three times the certainty equivalent returns. Clearly, fixed-point beliefs do a better job of managing tail risk while at the same time garnering higher profits at lower risk. Indeed, ex ante momentum returns under fixed-point beliefs are quite close to the ideal case of known capital allocations (Panel A) on virtually all dimensions. Certainty equivalent return differences between these two cases are below 0.1% for all values of risk aversion (γ) considered. Plot D.1 of Figure 1 suggests why: fixed-point beliefs recover the identity mapping seen with the known-capital case, as they must under rational expectations. This contrasts sharply with Plot C.1 (optimal linear beliefs). In short, crowding uncertainty imposes little cost when momentum investors form beliefs optimally, even with large σ_k^2. But that same crowding uncertainty can bring total devastation when investors ignore crowding concerns.

Plot C.2 indicates that ex ante momentum returns are fairly well behaved under optimal linear beliefs, despite being negative with some frequency. However, Plot D.2 (Fixed-point beliefs) indicate far more exacting defense against tail risk, as evidenced by the clear convergence away from negative ex ante momentum returns at high δ^E. This distinction between a constrained (linear) and unconstrained (fixed-point) belief mechanism is perhaps more strongly demonstrated in comparing Plots C.3 and D.3. Result 2 predicts that the momentum portfolio will always be subject to some ex ante crash risk with optimized linear beliefs, unless investors entirely refrain from momentum trading (leaving substantial profits on the table). From Plot C.3, this incipient tolerance of tail risk under optimal linear beliefs is evident at higher values of unanticipated momentum capital, where a tilt toward crashes remains at the most extreme values of unanticipated crowding. This incipient tail risk does not arise under fixed-point beliefs.

Overall, these simulation results align well with the predictions of Result 2, and motivate

Result 3 *Crowding induced crash risk in momentum returns is eliminated if and only if momentum investors rationally incorporate tail risk into their conditional beliefs of fundamental value, $E(\delta|f)$. However, momentum returns are negatively related to unanticipated momentum capital $k_M - Ek_M$ under all specifications of beliefs.*

Result 3 implies that an empirical test for crowding-related momentum crashes is in fact a test of the rationality of arbitrageurs' beliefs. If we empirically observe a positive impact of crowding on higher moments, those effects must be due to irrationality in beliefs rather than crowding *per se*.

Final observation. The slope in Plot D.2 (as well as C.2) is negative, which runs contrary to the intuition that rational, risk-averse momentum investors seek a less aggressive position at lower m, and a more aggressive position at higher m, as in Eq. (6). If so, why does Plot D.2 imply that they seek less aggressive positions when they rationally (i.e., correctly) anticipate high fundamental value (high δ^E)? The answer must lie with f since $E_M m = \delta^E - f$. That is, it must be that m is low when δ^E is high because f is also high. But f is directly determined by momentum investors' demands, leaving us with the contradictory inference that momentum investor demands are high when momentum investors trade less aggressively.

In fact, this is not a contradiction at all; it is precisely what one expects with a rational consideration of crowding effects. To see this it is important to recognize that demands = ϕk_M, where $\phi = \frac{\delta^E - f}{\gamma(\delta^V + \sigma_d^2)}$ reflects the optimized intensity of demands per dollar of momentum capital given f. It is this intensity coefficient that relates to m, not demands overall. If k_M is large, then f will also tend to be large, driving down $E_M m$. But then each dollar of k_M (i.e., the intensity coefficient) trades less aggressively, moderating the impact of k_M on f. While demands overall are higher, each individual dollar trades less aggressively and preferences align the latter with m. On the other hand, if k_M is small, then f will also tend to be small implying a high $E_M m$ and aggressive trading by each individual dollar of momentum capital. In sum, crowding (k_M) is the missing degree of freedom resolving the apparent contradiction.

Figure 2 decomposes demands along these lines to test this logic using the same simulations as Plots D.1-3. Expected momentum returns are indeed increasing in ϕ as evidenced in Plot (a). Plot (b) reports the negative relation between k_M and m, and Plot (c) confirms that higher demands are indeed associated with lower m.

[Insert Figure 2 near here]

4 Empirical section

We base our empirical analysis on quarterly holdings from the Thomson Reuters Institutional 13F database starting in Q1 of 1980 and ending in Q3 of 2015. Stock data are from CRSP using price and share adjustment factors (restricted to CRSP share code 10 and 11 and a listing on AMEX, NYSE or Nasdaq). Daily and monthly momentum returns are obtained from Kenneth French's online data library. The momentum return at time t is defined as the return of the winners (those in the top 10% of the distribution according to returns from months $t-12$ to $t-2$) minus the return of the losers (those in the bottom 10% of the same distribution), using NYSE cutoffs. Returns are value-weighted within each decile.

4.1 Crowding proxies

At the end of quarter q, 13F institution i has equity capital $K_{i,q} \equiv \sum_{j=1}^{J} P_{j,q} w_{i,j,q}$, where $P_{j,q}$ is the price of stock j and $w_{i,j,q}$ is shares held. We first define two preliminary variables used to identify momentum investors. The first is

$$Buy_{i,q} = \sum_{j=1}^{J} P_{j,q} \left(w_{i,j,q} - w_{i,j,q-1} \left(\frac{K_{i,q}}{K_{i,q-1}} - r_{i,p,q} \right) \right) \iota_{j,q}, \tag{16}$$

where $\iota_{j,q} = 0$ unless the stock is a winner ($= 1$) or a loser ($= -1$). The multiplier on $w_{i,j,q-1}$ adjusts beginning of period weights for estimated flow during the quarter, under the premise that this induces proportional trading in existing holdings. We also track slower-moving institutions:

$$BuyP1_{i,q} = \sum_{j=1}^{J} P_{j,q+1} \left(w_{i,j,q+1} - w_{i,j,q} \left(\frac{K_{i,q+1}}{K_{i,q}} - r_{i,p,q+1} \right) \right) \iota_{j,q}. \tag{17}$$

Notice that while 13F data do not contain short positions, our measures identify momentum investing via net buying of winner stocks. In addition, *Buy* and *BuyP*1 also account for net selling in loser stocks, which further helps to identify momentum investing when it is implemented as part of a more diversified portfolio strategy.

We identify a momentum-investing institution with an indicator function $\mathbb{1}$ for $Buy > 0$ or $BuyP1 > 0$, for either one or four consecutive quarters. Let $i = 1$ to M_q index momentum investors according to the chosen indicator. For example, using the 4 quarter buy indicator

$$M_q = \sum_{i=1}^{N_q} \mathbb{1}_{\sum_{l=0}^{3} \mathbb{1}_{Buy_{i,q-l}}=4},$$

where N_q is the total count of institutions in quarter q.

To motivate our proxies for crowding, from Eq. (6) write the total demand in quarter q from momentum-investing institutions as

$$\sum_{i=1}^{M_q} \text{Demand}_{i,q} = \sum_{i=1}^{M_q} \left(\frac{E_{M,q}\left[m+\epsilon\right]}{\gamma Var_{M,q}\left[m+\epsilon\right]} \overline{K_{M,q}} \right) \frac{K_{i,q}}{\overline{K_{M,q}}} = \phi_q M_q. \tag{18}$$

Here we assume that all momentum investors maintain the same reaction function to the observable f, so they have the same beliefs in a given quarter, with the bracketed expression for ϕ_q being the representative momentum investors' intensity of demands. This decomposition of demands suggests that the count of momentum investing institutions is the primary source of uncertainty in crowding. Thus, the first set of proxies for crowding measure the unobserved count of peer momentum investors (which we scale by the concurrent total count to make it fractional).

$$\text{Cnt\_1qrt}_q = \frac{1}{N_q} \sum_{i=1}^{N_q} \mathbb{1}_{Buy_{i,q}>0}, \qquad \text{Cnt\_4qrt}_q = \frac{1}{N_q} \sum_{i=1}^{N_q} \mathbb{1}_{\sum_{l=0}^{3} \mathbb{1}_{Buy_{i,q-l}}=4}, \tag{19}$$

$$\text{CntP1\_1qrt}_q = \frac{1}{N_q} \sum_{i=1}^{N_q} \mathbb{1}_{BuyP1_{i,q}>0}, \qquad \text{CntP1\_4qrt}_q = \frac{1}{N_q} \sum_{i=1}^{N_q} \mathbb{1}_{\sum_{l=0}^{3} \mathbb{1}_{BuyP1_{i,q-l}}=4}. \tag{20}$$

In addition to *Buy* and *BuyP*1, we use a third preliminary variable $Cap_{i,q} = \sum_{j=1}^{J} P_{j,q} w_{i,j,q} \iota_{j,q}$ in

$$\text{Cap\_1qrt}_q = \frac{\sum_{i=1}^{N_q} Cap_{i,q} \mathbb{1}_{Buy_{i,q}}}{\sum_{i=1}^{N_q} K_{i,q}}, \qquad \text{Cap\_4qrt}_q = \frac{\sum_{i=1}^{N_q} Cap_{i,q} \mathbb{1}_{\sum_{l=0}^{3} \mathbb{1}_{Buy_{i,q-l}}=4}}{\sum_{i=1}^{N_q} K_{i,q}}, \tag{21}$$

to proxy total demand, $\phi_q M_q$. We consider three time series specifications of the proxies in Eqs. (19)- (21), meant to capture anticipated, unanticipated, and variance in crowd measures. $\text{Crowd}_{q\text{-}1}$ is a generic reference to levels and ΔCrowd_q is a generic reference to changes. These proxy anticipated and unanticipated crowding, respectively. $\hat{\sigma}_{\text{Crowd}}$ is a generic reference to the expected volatility of the measure using a GARCH(1,1) specification, and is our proxy for crowding uncertainty. ΔCrowd_q is generally our primary variable of interest.

Table 2 provides summary statistics for the 13F data (in Panel A), the proxies for momentum investing (in Panel B), and momentum returns (in Panel C). From Panel A, only 22% (1414/6360) of institutions consistently follow a momentum strategy, defined as having at least 2/3 of the available quarters satisfying a four-quarter *Buy* or *BuyP*1 indicator. By contrast, Grinblatt et al. (1995) find that 77% of mutual funds are momentum investors. The difference is likely attributable to a difference in definition.[9]

[Insert Table 2 near here]

Also from Panel A, momentum institutions have a higher turnover (24% compared to 21%), manage more assets (2.46 billion versus 1.23 billion), and hold more stocks (213 stocks on average versus 123).

Table 2, Panel B provides descriptive statistics of the crowding variables. Not surprisingly, at a 1qtr horizon approximately 50% of institutional investors are classified as momentum investors in a given quarter. The 4qtr measure is more relevant, averaging 11.7% for Cnt_ and 10.2% for CntP1_, compared with the $0.5^4 = 6.25\%$ value implied under a null hypothesis of no momentum-trading

[9]Grinblatt et al. (1995) define momentum investors each quarter if they buy the winners and sell the losers as defined by the returns over that same quarter.

institutions. Also from Panel B, most crowding variables show strong persistence as captured by their coefficients in an AR(1) regression. Given this (and below) evidence of persistence, we estimate the volatility of crowd using residuals from an AR(1) regression using a GARCH(1,1) specification (Bollerslev, 1986).

Table 2, Panel C summarizes regressions of momentum returns using the Fama-French 3 factor model (abbreviated FF3) and a dynamic version of the same model (DFF3).[10] Grundy and Martin (2001) show that the momentum portfolio has strongly time-varying risk exposures due to its rapidly changing composition. In the DFF3 specification we include regressors with an interaction dummy variable that takes the value one if the factor has a positive return in the previous year and zero otherwise (D preface in the variable names). In unreported results we also find that momentum has substantial crash risk in our sample (high excess kurtosis with pronounced left-skewness), which includes the momentum crash of March-May 2009.

Table 3, Panel A considers the persistence of our momentum classifications in more detail. For the _4qtr measures, the probability of maintaining the current classification in the following quarter is 69% (64%). The probability of remaining a momentum-trading institution four quarters ahead is 29%, versus an unconditional probability of 11% and a probability of 9% for a current non-momentum trader. The results in the case of CntP1_ are similar. Thus, the _4qtr measures provide a meaningful identification of momentum-trading institutions.

[Insert Table 3 near here]

One possible concern is that investors' preference for certain sectors or investment styles make them trade persistently with (or against) momentum for several quarters in a row. This possibility is challenged by the rapidly changing composition of the momentum portfolio itself, as in Table 3. Winners have a 55% chance of remaining winners the following quarter, but at four quarters the

[10]We often make reference to FF3 or DFF3 models or residuals. In fact, in the case of returns regressions the dependent variable is the raw momentum return and the FF3 or DFF3 regressors are included as controls, and for the crash and volatility analysis the residuals from the FF3 or DFF3 models are used.

likelihood is only 16%, which is actually less than the 23% chance of becoming a loser. Persistence is higher with losers, with 31% retaining that classification after four quarters.

4.2 Crowding and conditional expected returns on the momentum factor

Table 4 shows the results of predictive regressions of momentum returns on the various crowding measures. All momentum trading measures are appropriately lagged (in this and subsequent tables) to ensure that there is no overlap between the measurement of the independent variable and the momentum return. For example, we use the change in Cnt_1qrt_q to predict momentum returns in the quarter $q+1$, and the change in CntP1_1qrt_q to predict momentum returns in $q+2$. As a control we include lagged realized volatility of momentum computed from squared daily momentum returns in the previous quarter. Barroso and Santa-Clara (2015) show that this strongly predicts (negatively) momentum returns.[11] Because computing the regressors requires up to six quarters of data, the regression sample begins in Q3 1981 and ends in Q4 2015.

[Insert Table 4 near here]

The model predicts that both anticipated crowding (proxied with $\text{Crowd}_{q\text{-}1}$) and unanticipated crowding (proxied with ΔCrowd_q) negatively relate to momentum returns. As predicted, the effect is stronger for unanticipated crowding (from Table 4, Panel A the difference is statistically significant at the 1% level using Cnt_4qtr and CntP1_4qtr).[12] However, statistical reliability is lost in Panel B using the noisier _1qtr measures. We generally do not see this relation with Cap_ measures, where the coefficient is insignificant in the ΔCrowd_q specification and positive—often reliably so—in the $\text{Crowd}_{q\text{-}1}$ specification.

This difference between the Cnt_ and Cap_ specifications, particularly with the anticipated crowding proxy $\text{Crowd}_{q\text{-}1}$, suggests that momentum investors adjust their demands in response to

[11] In unreported results we also controlled for the bear market states proposed by Cooper et al. (2004). Using this control in our sample period did not change our results.

[12] Unless otherwise noted the significance levels discussed refer to two-tailed tests even when the model provides a clear prediction for the sign of the coefficient.

crowding risk. This is the first of several pieces of evidence we provide that supports the rational belief-formation hypothesis over the naïve-beliefs alternative. We conjecture that rational momentum investors find it relatively difficult to accurately condition on the count N_q but relatively easy to condition on how intensely competitors will trade ϕ_q under observed market conditions.

Finally, from Table 4 the anticipated volatility of crowding, $\hat{\sigma}_{\text{Crowd}}$, is positively related to the expected return of momentum at the 1% level for one specification (CntP1_ and DFF3 returns), but the relation is generally insignificantly positive. A positive relation implies that uncertainty in the number of competing momentum investors inhibits participation in the strategy, but that inference is cloudy at best. In our later consideration of volatility in momentum returns (Table 7) we find evidence (again cloudy) consistent with this story.

4.3 Crowding and negative tail events in momentum returns

Of particular interest is the relation between unanticipated crowding and the pronounced left tail in the distribution of momentum returns. We assess this link with a probit analysis of tail probabilities using the _4qtr measures and both raw and DFF3 return specifications. We find a statistically reliably positive relation between $\Delta\text{Crowd}_\text{q}$ and the probability of a momentum return in the 10% left tail using Cnt_ and CntP1_ measures. This suggests that crowding indeed contributes to tail risk, but the evidence is weaker using 5% tails, perhaps because of small-sample difficulties.

[Insert Table 5 near here]

The p-values in square brackets from Table 5 provide the primary test of the table. The evidence of a mean shift in Table 4 suggests an overall shift in the distribution of momentum returns with crowding. As a result, even if crowding had no effect on the likelihood of extreme negative returns (relative to, say, the mean), the probit analysis might indicate tail risk. Thus, we investigate the effect of crowding on left tail probabilities using a bivariate probit analysis that considers the shift in probabilities for both the left and right tails. The null hypothesis is that any fattening observed in the left tail is caused by a downward shift in the mean, rather than an elevation in negative skew.

The Wald tests indicate that the impact of crowding on tail risk is largely due to a shift in mean returns rather than increased negative skew. While a few p-values are significant at conventional levels, overall the evidence against the null hypothesis of a symmetric effect is very weak. The relation between ΔCrowd_q using the Cap_ measure and left-tail probabilities is also positive, but not statistically reliable, as are the relations with $\text{Crowd}_{q\text{-}1}$. This implies that the relation between crowding and left-tail returns, to the extent there is a relation, is due to a unanticipated number of peers, not unanticipated trade intensity or the anticipated number of competitors.

We find no evidence that the GARCH measure of crowding volatility ($\hat{\sigma}_{\text{Crowd}}$) predicts left-tail events. However, the Wald tests for a differential effect on left versus right tails are suggestive, with five of eight p-values under 5% for the case of Cnt_ measures. Hence, while the conclusion is not without some ambiguity, the evidence in Table 5 generally finds little support for the hypothesis that arbitrageur crowding contributes to momentum crashes.

[Insert Figure 3 near here]

Figure 3 plots the time series of our crowding measures. The measures shown suggest the momentum strategy was indeed crowded during the internet bubble. Piazzesi and Schneider (2009) find similar evidence of increased trend following behavior during the housing bubble of 2007-2009 and argue that the actions of a small cluster of momentum investors can exert considerable influence on prices. On the other hand, no striking pattern is discernible before or during the major momentum crash of 2009. If anything, momentum investing by 13F institutions seems to have retracted prior to that crash.

4.4 Tail risk

While the notion of tail risk surely involves pronounced left skewness, to be meaningful it must also be accompanied by high volatility and excess kurtosis.[13] With this in mind we examine all

[13] A large kurtosis combined with left skewness is much more meaningful if volatility is also high. A low volatility directly reduces the denominator in these quantities inflating their values.

three moments of momentum returns in Table 6 by sorting calendar quarters based on our Crowd measures. We add sorts based on lagged realized volatility of momentum returns for comparison.

The evidence in Table 6 does not support crowding as a source of tail risk in momentum returns. Rather, momentum investors seem somewhat successful in avoiding it. Consider the ΔCrowd_q measures, our proxy for unanticipated crowding. Generally, ΔCrowd_q is statistically unrelated to subsequent values of each of the three contributors to tail risk; volatility, skewness, and kurtosis. If anything, the relation is negative. In particular, using the Cnt_ variables, both negative skew and excess kurtosis take on their most moderate values in the top tercile of ΔCrowd_q, and are much higher in the bottom tercile. Though not statistically significant, these differences certainly do not support the hypothesis that unanticipated crowding generates tail risk. For the anticipated crowding variable $\text{Crowd}_{q\text{-}1}$, particularly using Cnt_ measures, the results are likewise inconsistent with crowding-induced tail risk, except that here the relatively low risk associated with the high tercile of the crowding measure *is* statistically reliable.

Collectively, the results in Table 6 suggest that momentum investors are somewhat successful in avoiding momentum tail risk, i.e., they employ rational beliefs that endogenize the potential for crashes and thereby act to ameliorate that potential.

[Insert Table 6 near here]

In sharp contrast to the results conditioning on crowding measures, conditioning on lagged volatility in momentum returns positively predicts all three moments associated with tail risk. Returns following quarters with high volatility have an annualized volatility of 38.7%, versus 15.3% following low volatility. The difference between the two terciles has a t-statistic of 5.7. Both negative skew and excess kurtosis are statistically reliably more extreme in the high volatility tercile. This evidence is consistent with Barroso and Santa-Clara (2015) who find that a volatility-managed momentum strategy has much smaller crash risk than original momentum. The evidence in Table 6 supports their suggestion that the volatility managed strategy works because volatility clustering generates excess kurtosis in momentum returns.

This result offers an important insight. There is tail risk in our sample and it does relate to ex ante market conditions, however, crowding (particularly anticipated crowding) does not seem to be one of them. Indeed, the contrast in Table 6 between the predictability of volatility and the predictability of crowding variables (particularly the contrast in direction of predictability) strongly suggests that momentum investors form beliefs that incorporate tail risk. As in the theory, this has the effect of mitigating tail risk. Rampant momentum investing (as in the high tercile of Crowding) seems to identify a predictably stable environment whereas retraction from the momentum strategy (the low tercile of Crowding) seems to identify a toxic environment. Volatility seems to be at least one signal that rational arbitrageurs use to manage tail risk.

We agree with much of the literature that it is tempting to interpret the predictive power of volatility for momentum returns as indirect evidence that unanticipated crowding in the momentum strategy contributes to instability—and crashes—in momentum returns. Indeed, we embarked upon the project with just this perspective. However, direct examination of the hypothesis finds the opposite: optimizing momentum investors seem to identify the potential for crowding to destabilize and adjust their demands accordingly.

4.5 Crowding and the volatility of momentum returns

If volatility clustering in momentum returns were driven by the crowding of momentum investors rather than factors exogenous to arbitrageur actions, we should find that momentum volatility is predicted by crowding measures. To examine this hypothesis, Table 7 presents predictive regressions for realized volatility of momentum returns, computed from raw and risk adjusted (both static and dynamic) daily returns over the quarter.

[Insert Table 7 near here]

Consistent with the results in Table 6, Table 7 shows that lagged realized volatility has strong predictive power for subsequent volatility, with t-statistics between 6.7 and 8.5 across all regressions. The table also provides somewhat weak evidence that uncertainty about crowding, as proxied with $\hat{\sigma}_{\text{Crowd}}$, predicts positively the subsequent volatility of momentum returns. The coefficients have the predicted positive sign for all proxies and 7 out of 18 show significance at the 5% level, with a couple more at the 10% level. Thus, crowding uncertainty seems to contribute to risk in momentum returns.

The finding of a predictive role for $\hat{\sigma}_{\text{Crowd}}$ is consistent with the Table 4 evidence (albeit weak) that $\hat{\sigma}_{\text{Crowd}}$ predicts positively momentum returns. If momentum investors pull back from the strategy when $\hat{\sigma}_{\text{Crowd}}$ is high in the belief that $\hat{\sigma}_{\text{Crowd}}$ predicts risk, then a positive relation in Table 4 should follow. The forecasting power of $\hat{\sigma}_{\text{Crowd}}$ in Table 7 is stronger with the _1qtr measures. Presumably, stability and persistence in the _4qtr measures — an advantage with ΔCrowd_q and $\text{Crowd}_{q\text{-}1}$ — is a disadvantage when it comes to proxying uncertainty. This makes sense given the GARCH structure of the forecast and the very low resolution of a three-month observation on trading. We conjecture that this explains the overall weak evidence with the $\hat{\sigma}_{\text{Crowd}}$ measure.

Moving on to the ΔCrowd_q and $\text{Crowd}_{q\text{-}1}$ measures, Table 7 provides a fairly robust inference that crowding predicts negatively volatility in momentum returns. The coefficient estimate on both specifications (applied to Cnt_ measures) is statistically significant at the 5% level in 13 of 24 cases across Panels A and B, and is negative in all cases. This result is consistent with expectations of risk in the momentum strategy affecting institutions' willingness to participate in the strategy. Note that this interpretation implies that forward-looking institutions observe a wider information set than just lagged volatility in momentum returns, which we have controlled for.

The explanatory power of crowding measures in Table 7 pales in comparison to that of lagged realized volatility. This is perhaps not surprising, as lag dependent variables capture all persistent characteristics of the setting. Moreover, lag volatility is estimated using daily data whereas crowding measures are based on holdings data observed at a quarterly frequency. What the crowding measures have going for them is that they are explicit economic measures brought to bear on the data from theory. Lag dependent variables offer little insight beyond persistence in setting. We believe that this fact makes up for the shortcoming in predictive power.

4.6 Determinants of crowding

In this section we consider the dependence of crowding on lag returns and volatilty. Chabot et al. (2014) show in a comprehensive sample period of 140 years that the crash risk of momentum increases after periods of good recent returns in the strategy. Piazzesi and Schneider (2009) use survey data to study momentum investing in the US housing market and find a substantial increase in the number of momentum investors towards the end of the housing boom.

[Insert Table 8 near here]

Table 8 shows the results of regressions of Crowd_q on one-year momentum returns and one-year volatility of returns computed from daily observations, lagged at the indicated horizon. To ensure predetermined values for the _4qtr measures, we add one-year returns and volatility lagged five quarters to the analysis. From Table 8, the Cnt_4qtr specification of Crowd_q is significantly negatively related to one-year volatility lagged one quarter, but overlap in observation periods obfuscates the sequencing of events here (we have already seen from Table 7 that Cnt_ measures predict negatively future risk in the momentum strategy). However, from Table 8 we see that the Cnt_1qtr specification of Crowd_q reacts negatively to past volatility in momentum returns with a statistically reliable relation at the 1% level. Combining inferences from the anticipation and reaction analyses (i.e., Tables 7 and 8), we conclude that volatility in momentum returns is a primary determinant of crowding; that its impact is largely anticipatory; and that the response is fast (no reliable dependence of the Cnt_4qtr specification of Crowd_q on 5-quarter lagged volatility).

Table 8 also shows that one-year returns predict positively crowding in momentum. The coefficients on lagged returns from Table 8 are positive in all regressions and statistically significant at the 1% level in eight cases out of twelve. In the case of overlap between the estimation period for _4qtr measures and returns (i.e., top row), the relation is less positive due to the negative effect of crowding on returns. However, in the case of returns lagged five quarters, there is a reliable positive relation with the Cnt_ specifications, indicating that crowding by peer institutions reacts to past momentum returns. This is also seen in the _1qtr specifications with one-quarter lagged returns. The Cap_ specification relates positively to past returns only in the one quarter case.

Cooper et al. (2004) show that momentum returns are stronger in bull markets. That evidence supports the interpretation of momentum as partially caused by over-confident and self-attributing investors becoming particularly over-confident during bull markets (Daniel et al., 1998). In unreported results, we do not find any predictive power of market states for our measures of momentum crowding once controlling for the lagged returns and volatility of the strategy.

Our results are consistent with the momentum strategy becoming more crowded when its recent performance is good both in terms of high returns and low volatility. The volatility results suggest that forward-looking rational momentum investors successfully time risk in the strategy. That may be partly causal as exodus from the strategy potentially generates self-fulfilling risk. On the other hand, while chasing momentum returns is harder to rationalize, it also does not seem to be detrimental. In particular, the strategy returns do not show time-series autocorrelation and past one-year returns are not related to higher crash risk in our sample period.[14]

4.7 Capital versus crowd

We have argued that Cnt_ measures track the number of momentum investors and Cap_ measures track the intensity of their trade. In our final analysis we consider the two dimensions of crowding in a multivariate setting. As the intent is to summarize, we consider all three moments of momentum returns in a single table using the dynamic Fama-French model using the Cnt_ (rather than CntP1_) specification for the number of momentum-trading institutions.

Results for returns are presented in the first two columns of Table 9 for the _1qtr and _4qtr measure, respectively. In comparing the columns, it is clear that the more stable and persistent _4qtr measure does a better job of predicting momentum returns. This emphasizes the importance of identifying momentum-trading institutions rather than just aggregating institutional trading in

[14]In unreported results and as opposed to Chabot et al. (2014), we found no significant relation between past one-year momentum returns and crashes in probit regressions controlling for momentum's volatility.

momentum stocks. From the _4qtr column it is also clear that the *number* of momentum investors rather than the *intensity* of their trading (as in Eq. (18)) is most relevant in identifying pricing effects of crowding. This is further supported by the strength of the relation for ΔCrowd_q, which we argue proxies for unanticipated crowding. Note also that the positive relation of $\hat{\sigma}_{\text{Crowd}}$ in predicting momentum returns is here statistically significant at the 1% level.

[Insert Table 9 near here]

In contrast with the negative predictive relation of Cnt_ measures for returns, the Cap_ measures are positively related to future returns, significantly so with the $\text{Crowd}_{q\text{-}1}$ specification. Because these regressions estimate marginal effects and the marginal effect of Cap_ captures the intensity of optimized trade, this evidence suggest that the number of institutions is the more relevant dimension for crowding effects, not the intensity of trade.

The tail-risk results in the third and fourth columns of Table 9 largely confirm the results in Table 5. Unanticipated crowding increases the probability of left-tails events with Cnt_ specifications but not Cap_ specifications. However, the relations are mostly due to a shift in mean rather than more pronounced left skew. The Wald tests for a differential effect on the left and right tails is no where significant.[15] Volatility results likewise confirm the earlier analysis (Table 7).

4.8 Comparison with return-based measures of crowding

Lou and Polk (2013) propose comomentum, a measure of abnormal co-movement of stocks in the momentum portfolio, as a proxy for crowding. In support of this proposition, they document a positive relation between comomentum and aggregate institutional ownership of the winners portfolio. Huang (2015) argues that the momentum gap, defined as the cross-sectional dispersion of formation period returns, also proxies for crowding. He supports this by showing that it is related to the difference in institutional ownership for winner versus loser portfolios. Both studies find that the indirect, returns-based proxy for crowding negatively relates to momentum returns. Finally,

[15] In two of twelve cases the test is significant at a 10% level.

volatility in momentum could also be hypothesized to arise from investor crowding, potentially representing a third returns-based proxy.

We have already seen that the relation between volatility and institutional crowding is more consistent with investors using volatility as a signal to avoid tail risk than volatility being caused by crowding. The question we address here is how other returns based proxies relate to our direct, trading-based measures of institutional crowding. We focus on the momentum gap due to its simplicity; because it is a strong predictor of risk and return for momentum; and because of its proximity to the theory.

[Insert Table 10 near here]

Table 10 mirrors Table 6, except that the focus is momentum gap and momentum gap orthogonalized to our crowding measures. We explore the conjecture that momentum gap's predictability stems from institutional crowding. If so, orthogonalizing should attenuate its predictability. In each column, we rank all months in our sample into terciles according to the value of a sorting variable in the last available quarter. In the first column the ranking variable is the momentum gap. Consistent with Huang (2015), we find that a high momentum gap forecasts higher volatility, negative skewness, and excess kurtosis; all with statistical significance at the 1% level.

Next we orthogonalize momentum gap with respect to measures of institutional crowding (denoted gap$^{\perp}$). In addition to levels of Cnt_4qtr, CntP1_4qtr, and Cap_4qtr, we consider ΔMom Inst from Huang (2015), defined as the percentage difference in aggregate institutional ownership between past winners and losers; and Win Inst from Lou and Polk (2013), defined as the aggregate institutional ownership of the winner decile. Columns 2 to 6 show that gap$^{\perp}$ retains substantially all of the predictive power of the momentum gap itself. Overall, the evidence in columns 2 to 6 shows that while the momentum gap is a strong predictor of the performance of the strategy, its predictability does not appear to be attributable to crowding.

Finally, in the last column of Table 10 we orthogonalize the momentum gap to the volatility measure used as a control in many of our empirical exercises. Since the momentum gap is defined as the inter-quartile range of the return distribution for stocks in a given formation period, it is a measure of (cross-sectional) dispersion in returns that should be closely related to volatility. Indeed, the correlation between momentum gap and volatility is 0.73 in our sample period. Nevertheless, gap$^{\perp}$ remains a reliable predictor of volatility, averaging 35.6% following high gap$^{\perp}$ versus 21.0% following low gap$^{\perp}$ with a t-statistic for the difference of 3.2. This suggests that momentum gap captures different information from that contained in volatility. However, volatility largely strips momentum gap of its forecast power for higher moments of momentum returns. While the point estimates imply more negative skewness and higher kurtosis with high gap$^{\perp}$, the differences with low gap$^{\perp}$ are not statistically reliable.

Momentum gap relates to tail risk. In principal, that relation could be a reflection of crowded trades, or some other destabilizing shock that drives prices far apart in the formation period only to reverse badly in the evaluation period. Our evidence suggests the latter, a conclusion mirrored with our results for volatility. Both volatility and momentum gap predict tail risk *despite* the behavior of institutions, not because of their behavior.

5 Conclusion

We provide a model based on momentum investors seeking to exploit inference from past returns regarding the incomplete assimilation of informed investors' private signals of value. The model is similar to Stein (2009) in setting, but our analysis differs in the emphasis that we place on rational beliefs. Our primary result is that predictions of destabilizing effects from unanticipated crowding require a naïve, linear specification of beliefs, where momentum investors do not adequately account for the potential destabilizing effect of crowding on prices. With rational (generally nonlinear) beliefs directly computed as a fixed-point solution to the market clearing equation, the potential destabilizing effects of crowding are internalized into demands, mitigating the feedback behavior that would otherwise lead to destabilized prices. In short, our theory shows that crowding is not a viable source or momentum crashes, only crowding with naïve beliefs is.

Our empirical contribution is twofold. First, we directly examine proxies for momentum trading by institutional investors, in contrast to much of the literature which focuses on indirect inferences of crowding from return covariances or volatility. Second, we directly examine the implications of optimal versus naïve beliefs for patterns in momentum trading patterns.

Across the empirical analyses, we consistently find evidence of crash-avoidance behavior rather than destabilizing feedback trading. Consistent with our theory under rational beliefs, we find no evidence that crowding by momentum investors drives up the higher moments of momentum returns (that is, causes crashes), despite a clear impact on the first moment of returns. We do find that past volatility in momentum returns identifies crashes, as in prior studies. But we also find that momentum investors both control for this result as well as condition on other sources to anticipate and back away from periods of instability.

A Existing literature

Our paper is related to the empirical and theoretical literature on momentum. Momentum was initially documented for US stock returns (Levy, 1967; Jegadeesh and Titman, 1993) and has since been documented for stock returns in most countries (Rouwenhorst, 1998) and across asset classes (Asness et al., 2013). Besides its very high average returns, momentum carries significant downside risk or negative skewness in the form of occasional large crashes (Daniel and Moskowitz, 2016). Existing research also shows that institutional investors are momentum traders, i.e., tilt their portfolios towards momentum stocks (Grinblatt et al., 1995; Lewellen, 2011; Edelen et al., 2016). Our paper contributes to this literature by directly examining whether uncertain institutional participation in the momentum strategy is the source of higher-moment return characteristics.

A recent empirical literature examining the time series properties of momentum finds results broadly consistent with an over-reaction explanation of the effect. The premium is stronger in periods of bull markets (Cooper et al., 2004), high liquidity (Avramov et al., 2016), high sentiment (Antoniou et al., 2013), and low market volatility (Wang and Xu, 2015). Hillert et al. (2014)'s finding that momentum is more pronounced in firms with more media coverage also supports an over-reaction interpretation, as does the evidence in Edelen et al. (2016) regarding institutional purchases in the portfolio-formation period. As a whole, this evidence suggests that crowding plausibly explains the higher-moment characteristics of momentum.

On the other hand, the momentum premium is stronger in stocks experiencing frequent but small price changes that are less likely to attract attention (Da et al., 2014) or those characterized by small trades of investors under-reacting to past returns (Hvidkjaer, 2006). Also there is recent evidence that momentum is somehow explained by improvements in firm fundamentals (Novy-Marx, 2015; Sotes-Paladino et al., 2016; DeMiguel et al., 2017). This evidence suggests momentum investors exploit under-reaction and as such (exogenous increases in) crowding should reduce its premium.

The related theoretical literature on momentum offers theories based on institutional investors and fund flows (Vayanos and Woolley, 2013) or behavioral biases such as over-reaction / self-attribution (see, e.g., Daniel et al., 1998; Barberis et al., 1998) or information externalities and gradual diffusion of information (see, e.g., Stein, 1987; Hong and Stein, 1999; Andrei and Cujean, 2017). Our work is most closely related to the latter branch of the literature.

Our model builds on the information externality that the actions of unanticipated momentum investors impose on their peers. Thus, it is closest in development to Stein (2009), but follows in a long line of research relating to arbitrageur information coordination and externalities. This literature dates to Stein (1987) who characterizes the externality, and Scharfstein and Stein (1990) and Froot et al. (1992) who relate it to herding behavior. Hong and Stein (1999) relate the externality to persistence and reversal patterns in returns. A related branch of the literature identifies the positive feedback trading of momentum investors as a source of destabilizing noise in prices, e.g., De Long et al. (1990a,b).

Kondor and Zawadowski (2015) study whether the presence of more arbitrageurs improves welfare in a model of capital reallocation. Trades in the model can become crowded due to imperfect information, but arbitrageurs can also devote resources to learn about the number of earlier entrants. They find that if the number of arbitrageurs is high enough, more arbitrageurs do not change capital allocations, but decrease welfare due to costly learning.

Related empirical research includes Hanson and Sunderam (2014) who construct a measure of the capital allocated to momentum and the valuation anomaly (book-to-market or B/M) using short-interest. They find some evidence that an increase in arbitrage capital has reduced the returns on B/M and momentum strategies. In addition, Lou and Polk (2013) proxy for momentum capital with the residual return correlations in the short and long leg of the momentum strategy and find that momentum profits are lower and crashes more likely in times of higher momentum capital. While our analysis uses a different approach and insights in proxying for momentum capital, our result on unanticipated momentum capital and momentum returns is generally consisted with their finding, but we cannot attribute momentum's crashes to crowding. Finally, Huang (2015) proposes a momentum gap variable, which is defined as the cross sectional dispersion of formation period returns. He shows that this measure predicts momentum returns and crashes, and argues that this is consistent with Stein (2009)'s crowded trade theory. Throughout our analysis, we control for momentum's past volatility, which has a correlation of 0.73 with the momentum gap measure. We also verify in Section 4.8 that momentum gap's predictive power for crash risk is unrelated to various institutional measures of momentum crowding. This corroborating our finding that momentum's crashes are not explained by crowded trades of institutions.

We go beyond the usual focus on first moments to study the determinants of the risk of momentum. This relates our work to a recent strand of literature focusing on the predictability of the moments of momentum. Barroso and Santa-Clara (2015) show that the volatility of momentum is highly predictable and it is a useful variable to manage the risk of the strategy. Daniel and Moskowitz (2016) argues the crash risk of momentum is due to the optionality effect of the losers portfolio that resembles an out-of-the-money call option after extreme bear markets. Jacobs et al. (2015) examine the expected skewness of momentum as a potential explanation of its premium. They propose an enhanced momentum strategy but find that managing its risk results in a performance hard to reconcile with a premium for skewness. Grobys et al. (2016) find industry momentum has different risk properties from standard momentum but shows similar gains from risk management. Our results address the question of whether investors condition their exposure to momentum using this new-found predictability. Consistent with the economic case for managing the risk of momentum, we find the subset of momentum investors shrinks after periods of high volatility.

B Development of momentum portfolio

Stocks are indexed by j. Each pays a discrete dividend X_j which evolves according to

$$\log\left(X_j/X_{j,0}\right) = \chi + \iota_j \frac{d}{2},$$

where $X_{j,0}$ is the beginning of period dividend, χ is a random zero-mean innovation common to all stocks with variance σ_χ^2 that generates the market return; the indicator ι_j selects the momentum portfolio, taking on the value 1 or -1 for 10% of all stocks (in each leg); and d generates the differential return on the two groups of stocks, with variance σ_d^2 and mean δ where δ is the private information in the market. All investors know σ_d^2 and σ_χ^2. Let $\boldsymbol{X}$ denote the vector of all stocks' dividends, which is unknown prior to the end of the period, at which point it becomes known to all. We refer to stocks with $\iota_j = 1\,(-1)$ as winner (loser) stocks.

At the beginning of the period information is symmetric, hence each investor holds the market portfolio. This results in a vector of public-information valuations $\boldsymbol{P}_0 = \boldsymbol{X}_0/r$ where r denotes an unmodelled required return on the market portfolio. At some intermediate time within the period (portfolio formation date) a subset of investors managing beginning-of-period capital K_I observes δ and $\boldsymbol{\iota}$ and trades. This trading identifies the momentum portfolio by generating a formation period return on winner and loser stocks via informed investors' demands. Informed investors expect an end of period price increase of $e^{\frac{1}{2}\left(\delta+\sigma_\chi^2\right)+\frac{1}{8}\sigma_d^2}$ for each winner stock and a price decrease of $e^{\frac{1}{2}\left(-\delta+\sigma_\chi^2\right)+\frac{1}{8}\sigma_d^2}$ for each loser stock. Neutral stocks experience no trading, because all market participants maintain the same expected end of period dividend increase of $e^{\frac{\sigma_\chi^2}{2}}$ for such stocks.

Uninformed (i.e., momentum and counterparty) investor begin with homogeneous expectations of dividends for all stocks, equal to their beginning of period values. Thus, the returns on each winner; loser; and neutral stock are homogeneous within type. Moreover, by presuming the same average information signal on winner stocks ($\delta/2$) as loser stocks ($-\delta/2$), the long-short portfolio of winners minus losers (i.e., the momentum portfolio), can be treated as a single asset. The information signal on this long-short portfolio, δ, parameterizes the private information in the market. When we refer to ex ante momentum returns, we mean the expectation of the post-formation date return conditional on δ and market-clearing prices on the formation date, denoted m in the paper.

C Derivation of Eq. (6)

First notice that solving Eq. (5) is equivalent to solving each of the following (presuming $\gamma > 1$)

$$\max_{\varsigma} \quad \frac{K_{type}^{1-\gamma}}{1-\gamma} \cdot E\left[e^{(1-\gamma)\left(r_f+\log\left(1+\varsigma\left(e^{r_p-r_f}-1\right)\right)\right)}\right] \quad \Leftrightarrow \quad \min_{\varsigma} \quad \log E\left[e^{(1-\gamma)\log\left(1+\varsigma\left(e^{r_p-r_f}-1\right)\right)}\right],$$

where ς is the weight on the portfolio of risky asset and r_p its log return, and r^f is the log risk-free rate. Second, to solve for the fraction of wealth invested in the risky portfolio, we follow Section 2.1.1 in Campbell and Viceira (2002, Internet Appendix) and approximate the function $g\left(r_p - r_f\right) = \log\left(1 + \varsigma\left(e^{r_p-r_f} - 1\right)\right)$ using a second-order Taylor expansion around 0:

$$\begin{aligned} g\left(r_p - r_f\right) &\cong \log(1) + \frac{\varsigma e^0}{1+\varsigma\left(e^0-1\right)}\left(r_p - f\right) + \frac{1}{2}\frac{\varsigma\left[e^0\left(1+\varsigma\left(e^0-1\right)\right) - \varsigma e^{2\cdot 0}\right]}{\left(1+\varsigma\left(e^0-1\right)\right)^2}\left(r_p - f\right)^2, \\ &\cong \varsigma\left(r_p - f\right) + \frac{1}{2}\left(\varsigma - \varsigma^2\right)\sigma^2, \end{aligned} \tag{C.1}$$

where $\left(r_p - f\right)^2$ is replaced with its expected value σ^2. Using (C.1), we can then rewrite the maximization problem to

$$\begin{aligned} &\min_{\varsigma} \quad \log E\left[\exp\left[\frac{1}{2}\left(\varsigma - \varsigma^2\right)(1-\gamma)\sigma_p^2\right] \cdot \exp\left[\varsigma(1-\gamma)\left(r_p - r_f\right)\right]\right], \\ \Leftrightarrow &\min_{\varsigma} \quad \frac{1}{2}\left(\varsigma - \varsigma^2\right)(1-\gamma)\sigma_p^2 + \varsigma(1-\gamma)\left(\mu_p - r_f\right) + \frac{1}{2}\varsigma^2(1-\gamma)^2\sigma_p^2, \\ \Leftrightarrow &\max_{\varsigma} \quad \varsigma\left(\mu_p - r_f + \frac{1}{2}\sigma_p^2\right) - \frac{1}{2}\varsigma^2\gamma\sigma_p^2, \end{aligned}$$

which has solution

$$\varsigma = \frac{\mu_p - r_f + \frac{1}{2}\sigma_p^2}{\gamma\sigma_p^2}.$$

To proceed, we will assume that log returns and arithmetic returns are similar such that $\mu_p - r_f + \frac{1}{2}\sigma_p^2 \cong e^{\mu_p - r_f} \cong \mu_p - r_f$. We then determine μ_p and σ_p^2 in the context of a portfolio comprised of the market investment plus a long-short momentum investment. Because the momentum portfolio is self-financing, feasible combinations of the market portfolio and the momentum portfolio are given by the weight vector $\boldsymbol{w}' = \begin{bmatrix} 1 & w_m \end{bmatrix}$, i.e., hold the market portfolio plus a proportionate long-short momentum overlay w_m. The optimal risky portfolio is then that choice of w_m that solves the constrained optimization

$$\min_{\boldsymbol{w}} \quad \frac{\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}}{2}, \qquad \text{s.t.} \qquad \boldsymbol{\mu}'\boldsymbol{w} = r^* - r_f,$$

using weights $\boldsymbol{w}' = \begin{bmatrix} 1 & w_m \end{bmatrix}$, where

$$\boldsymbol{\mu} = \begin{bmatrix} r - r_f \\ E_{type}[m + \epsilon] \end{bmatrix}, \qquad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_\chi^2 & 0 \\ 0 & Var_{type}[m + \epsilon] \end{bmatrix},$$

and $r^* - r_f$ is a target return premium that traces out the efficient frontier (recall r is the required return on the market portfolio). This has solution

$$w_m = \frac{E_{type}[m + \epsilon] \Big/ Var_{type}[m + \epsilon]}{\left(r - r_f\right) \Big/ \sigma_\chi^2}. \tag{C.2}$$

Using (C.2), the parameters of the optimal risky portfolio are

$$\mu_p - r_f = \begin{bmatrix} r - r_f & E_{type}[m + \epsilon] \end{bmatrix} \begin{bmatrix} 1 \\ w_m \end{bmatrix} = r - r_f + w_m E_{type}[m + \epsilon],$$

and

$$\sigma_p^2 = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} = \sigma_\chi^2 + w_m^2 Var_{type}\left[m + \epsilon\right] = \frac{\sigma_\chi^2}{r - r_f}\left(r - r_f + w_m E_{type}\left[m + \epsilon\right]\right).$$

Taking the ratio gives

$$\varsigma = \frac{r - r_f}{\gamma \sigma_\chi^2}. \tag{C.3}$$

Combining (C.2) and (C.3),

$$Demand = w_m \varsigma K_{type} = \frac{E_{type}\left[m + \epsilon\right]}{\gamma Var_{type}\left[m + \epsilon\right]} K_{type}.$$

D Negative market-clearing price for momentum portfolio

In the case of $k_M > \lambda_E$, the demand of momentum investors (RHS Eq. (11)) increases with a positive f faster than the LHS supply can keep up with, implying an increasingly large buying imbalance as f rises (depicted in the third panel of Figure 1). This again suggests that momentum investors buy up to their capacity, leading to a subsequent momentum crash.

However, when $k_M > \lambda_E$ there is also a (finite) negative value for f that clears the market. While we discount this equilibrium as implausible, we note that even here the contrary pricing of winner and loser stocks implies a substantial negative momentum return, because the formation-period 'winners' are actually the fundamental losers, and vice versa.

It is not clear how this $f < 0$ equilibrium could be found, because informed investors presumably seed formation-period returns with *buying* of the momentum portfolio (and an initially positive f). Nevertheless, it is a call auction and if they were to bizarrely trade contrary to their private information, seeding a negative value for f, then they might be lucky enough to induce momentum investors into selling (buying) so much winner (loser) stock that their bizarre trade is preferred.

E Solution to the fixed-point problem

Let

$$F : (\delta, k_I, k_M) \to f = \frac{1}{D}\left(\delta k_I + \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right),$$

which characterizes market clearing as in (7). We map the primitive random variables into

$$\begin{pmatrix} \delta \\ k_M \\ k_I \end{pmatrix} \to \begin{pmatrix} \delta \\ k_M \\ F(\delta, k_M, k_I) \end{pmatrix}.$$

Next, we need

$$\mid \mathrm{J} \mid = \det \begin{pmatrix} \frac{\partial \delta}{\partial \delta} & \frac{\partial \delta}{\partial k_M} & \frac{\partial \delta}{\partial k_I} \\ \frac{\partial k_M}{\partial \delta} & \frac{\partial k_M}{\partial k_M} & \frac{\partial k_M}{\partial k_I} \\ \frac{\partial F^{-1}}{\partial \delta} & \frac{\partial F^{-1}}{\partial k_M} & \frac{\partial F^{-1}}{\partial k_I} \end{pmatrix} = \frac{D}{\delta}.$$

Following a standard result (see, e.g., Theorem 2 in Section 4.4 of Rohatgi and Saleh, 2000), the density is then given by

$$\begin{aligned} p_3(\delta, k_M, f) &= g(\delta)\, h\left(k_M, F^{-1}\right) \mid \mathrm{J} \mid \\ &= g(\delta)\, h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_d^2}} k_M\right)\right) \frac{D}{\delta}. \end{aligned}$$

References

D. Abreu and M. K. Brunnermeier. Bubbles and crashes. *Econometrica*, 71(1):173–204, 2003.

T. G. Andersen, T. Bollerslev, F. X. Diebold, and H. Ebens. The distribution of stock return volatility. *Journal of Financial Economics*, 61:43–76, 2001.

D. Andrei and J. Cujean. Information percolation, momentum and reversal. *Journal of Financial Economics*, 123(3):617 – 645, 2017.

C. Antoniou, J. A. Doukas, and A. Subrahmanyam. Cognitive dissonance, sentiment, and momentum. *Journal of Financial and Quantitative Analysis*, 48(1):245–275, 2013.

C. S. Asness, T. J. Moskowitz, and L. H. Pedersen. Value and momentum everywhere. *The Journal of Finance*, 68(3):929–985, 2013.

D. Avramov, S. Cheng, and A. Hameed. Time-varying liquidity and momentum profits. *Journal of Financial and Quantitative Analysis*, 51(6):1897–1923, 2016.

N. Barberis, A. Shleifer, and R. Vishny. A model of investors sentiment. *Journal of Financial Economics*, 49:307–343, 1998.

P. Barroso and P. Santa-Clara. Momentum has its moments. *Journal of Financial Economics*, 116 (1):111–120, 2015.

T. Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307 – 327, 1986.

J. Y. Campbell and L. M. Viceira. *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Oxford University Press, 2002.

B. Chabot, E. Ghysels, and R. Jagannathan. Momentum trading, return chasing, and predictable crashes. NBER Working Paper, 2014.

M. J. Cooper, R. C. Gutierrez, and A. Hameed. Market states and momentum. *The Journal of Finance*, 59(3):1345–1365, 2004.

Z. Da, U. G. Gurun, and M. Warachka. Frog in the pan: Continuous information and momentum. *Review of Financial Studies*, 22(7):2171–2218, 2014.

K. Daniel and T. J. Moskowitz. Momentum crashes. *Journal of Financial Economics*, 122(2):221 – 247, 2016.

K. Daniel, D. Hirshleifer, and A. Subrahmanyam. Investor psychology and security market under- and overreactions. *Journal of Finance*, 53:1839–1885, 1998.

J. B. De Long, A. Shleifer, L. H. Summers, and R. J. Waldmann. Noise trader risk in financial markets. *Journal of Political Economy*, 98(4):703–738, 1990a.

J. B. De Long, A. Shleifer, L. H. Summers, and R. J. Waldmann. Positive feedback investment strategies and destabilizing rational speculation. *The Journal of Finance*, 45(2):379–395, 1990b.

V. DeMiguel, A. Martin-Utrera, F. J. Nogales, and R. Uppal. A portfolio perspective on the multitude of firm characteristics. Working Paper SSRN, 2017.

R. M. Edelen, O. S. Ince, and G. B. Kadlec. Institutional investors and stock return anomalies. *Journal of Financial Economics*, 119(3):472 – 488, 2016.

K. A. Froot, D. S. Scharfstein, and J. C. Stein. Herd on the street: Informational inefficiencies in a market with short-term speculation. *The Journal of Finance*, 47(4):1461–1484, 1992.

M. Grinblatt, S. Titman, and R. Wermers. Momentum investment strategies, portfolio performance, and herding: A study of mutual fund behavior. *The American Economic Review*, 85(5):1088–1105, 1995.

K. Grobys, J. Ruotsalainen, and J. J. Äijö. Risk-managed industry momentum and momentum crashes. Working Paper SSRN, 2016.

B. D. Grundy and J. S. M. Martin. Understanding the nature of the risks and the source of the rewards to momentum investing. *The Review of Financial Studies*, 14(1):29, 2001.

S. G. Hanson and A. Sunderam. The growth and limits of arbitrage: Evidence from short interest. *Review of Financial Studies*, 27(4):1238–1286, 2014.

A. Hillert, H. Jacobs, and S. Müller. Media makes momentum. *The Review of Financial Studies*, 27(12):3467–3501, 2014.

H. Hong and J. C. Stein. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of Finance*, 54:2143–2184, 1999.

S. Huang. The momentum gap and return predictability. *Working Paper SSRN*, 2015.

S. Hvidkjaer. A trade-based analysis of momentum. *Review of Financial Studies*, 19(2):457–491, 2006.

H. Jacobs, T. Regele, and M. Weber. Expected skewness and momentum. Working Paper SSRN, 2015.

N. Jegadeesh and S. Titman. Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance*, 48(1):65–91, 1993.

P. Kondor and A. Zawadowski. Learning in crowded markets. Working Paper Central European University, 2015.

R. A. Levy. Relative strength as a criterion for investment selection. *The Journal of Finance*, 22 (4):595–610, 1967.

J. Lewellen. Institutional investors and the limits of arbitrage. *Journal of Financial Economics*, 102(1):62 – 80, 2011.

D. Lou and C. Polk. Comomentum: Inferring arbitrage activity from return correlations. Working Paper London School of Economics, 2013.

R. Novy-Marx. Fundamentally, momentum is fundamental momentum. Working Paper 20984, National Bureau of Economic Research, 2015.

M. Piazzesi and M. Schneider. Momentum traders in the housing market: Survey evidence and a search model. *American Economic Review*, 99(2):406–11, 2009.

V. K. Rohatgi and A. K. M. E. Saleh. *Multiple Random Variables*, chapter 4, pages 102–179. John Wiley & Sons, Inc., 2000.

K. G. Rouwenhorst. International momentum strategies. *The Journal of Finance*, 53(1):267–284, 1998.

D. S. Scharfstein and J. C. Stein. Herd behavior and investment. *Amercian Economic Review*, 80 (3):465–479, 1990.

J. M. Sotes-Paladino, G. J. Wang, and C. Y. Yao. The value of growth: Changes in profitability and future stock returns. Working Paper SSRN, 2016.

J. C. Stein. Informational externalities and welfare-reducing speculation. *Journal of Political Economy*, 95(6):1123–1145, 1987.

J. C. Stein. Presidential address: Sophisticated investors and market efficiency. *The Journal of Finance*, 64(4):1517–1548, 2009.

D. Vayanos and P. Woolley. An institutional theory of momentum and reversal. *Review of Financial Studies*, 26(5):1087–1145, 2013.

K. Q. Wang and J. Xu. Market volatility and momentum. *Journal of Empirical Finance*, 30:79–91, 2015.

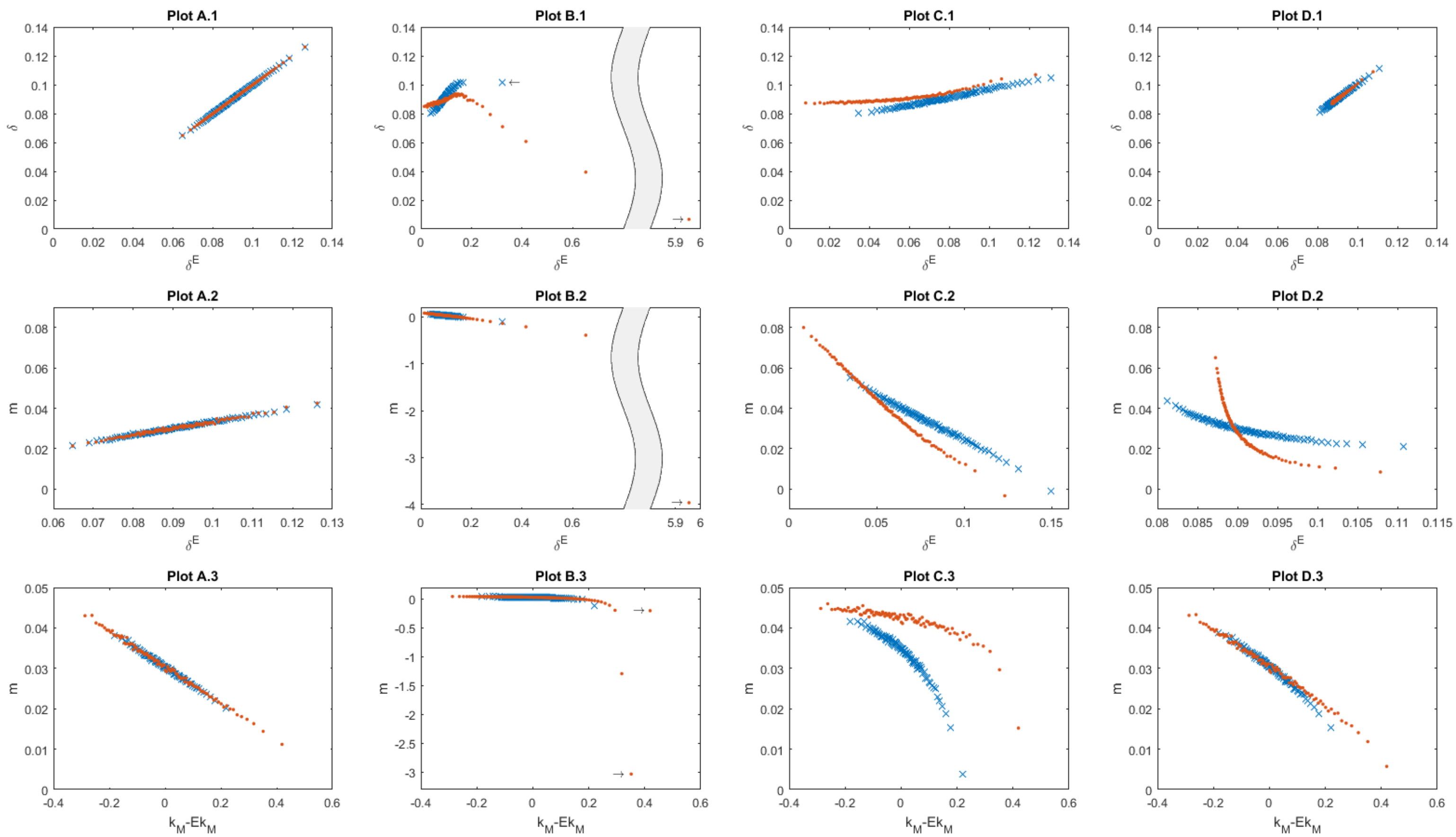
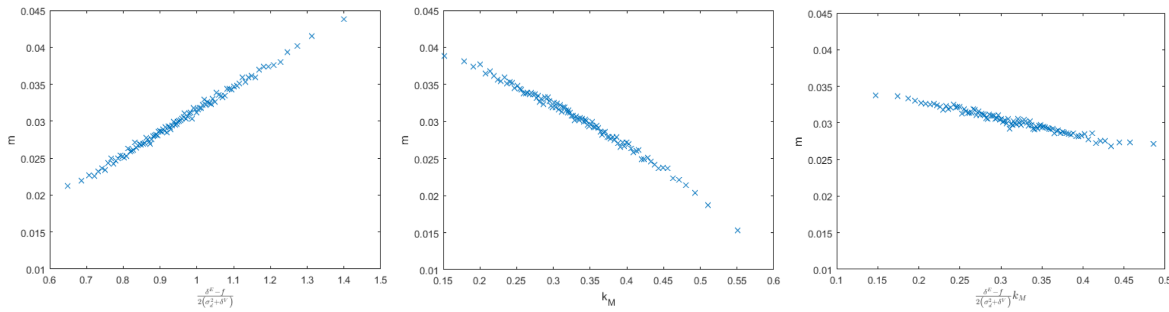


Figure 1: Beliefs and expected momentum returns in simulations

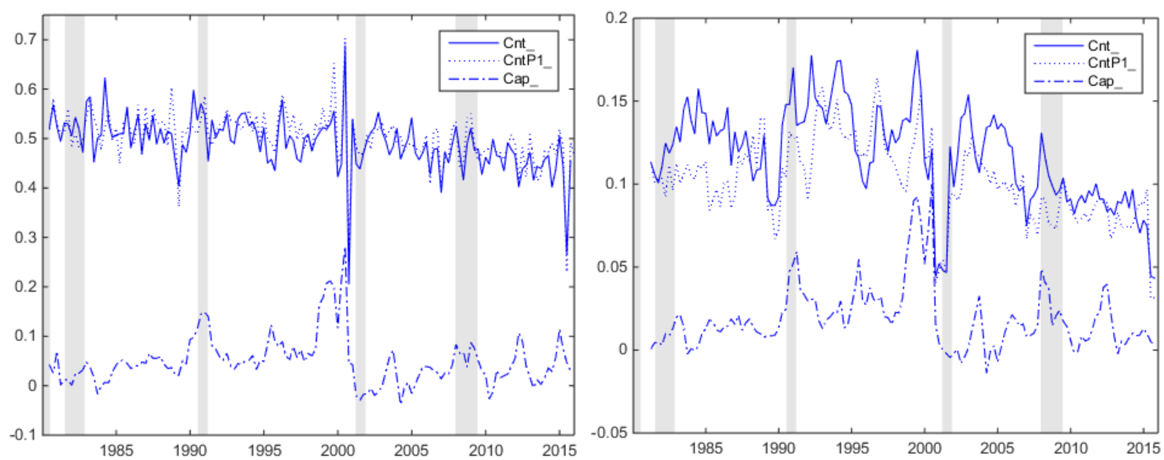
The simulations use 100,000 independent random draws of $\{k_I, k_M, \delta\}$, where k_I and k_M are informed and momentum capital, respectively, and δ is the signal of differential fundamental value for winners minus losers. k_I and k_M (and k_C) follow a Dirichlet distribution with concentration parameters $\alpha_i = 12$ (blue Xs) and 3 (red dots), and $Ek_I = Ek_M = 1/3$. δ follows a lognormal distribution with $\mu = -2.405$ and $\sigma = 0.125$ implying an average δ of 9.1% with standard deviation of 1.14%. The market clearing formation period return f is solved for each $\{k_I, k_M, \delta\}$ pair by iteration using different specifications for momentum traders' beliefs δ^E: linear beliefs with known capital shares in Plots A.1-3, naïve linear beliefs in Plots B.1-3, optimal linear beliefs in Plots C.1-3, and fixed-point beliefs in Plots D.1-3. The expected momentum return is then $m = \delta - f$. The triplets $\{\delta, m, k_M\}$ are ranked into 100 equally populated bins according to the horizontal-axis variable, and the plots represent the averages for the indicated variables within these bins.



(a) beliefs and preferences (b) capital (c) total demand

Figure 2: Momentum traders' demands and expected momentum returns: decomposition

This figure builds on the fixed-point simulations of Plot D.1-3 in Figure 1 with concentration parameters $\alpha_i = 12$, and decomposes the relation between momentum investors' aggregate demands and expected momentum returns. Panel (a) plots the within-bin average expected momentum returns versus momentum investors' demand due to beliefs and preferences (for $\gamma = 2$), $\frac{\delta^E - f}{2(\sigma_d^2 + \delta^V)}$, where averages are taken as in Figure 1 based on the horizontal-axis variable. Panel (b) and (c) are constructed in the same fashion, and plot average expected momentum returns, m, versus k_M and total momentum investor demands, $\frac{\delta^E - f}{2(\sigma_d^2 + \delta^V)} k_M$, respectively.



(a) one quarter (b) four quarters

Figure 3: Measures of crowding

Panel (a) and (b) report the _1qrt and _4qrt crowding measures, respectively, constructed with 13F holdings data in the period from 03/1980 to 09/2015. The shaded areas denote NBER recessions.

Table 1: Momentum returns in simulations

The table reports unconditional return statistics for the simulations described in the caption of Figure 1. The top part contains the descriptive statistics of expected momentum returns across all simulations. Mean, stdev, skew, kurt, min and max refer to average, standard deviation, skewness, kurtosis, minimum, and maximum, respectively. Panel A reports the results for the linear beliefs and known capital case, Panel B is the case with naïve beliefs, Panel C with optimal linear beliefs, and Panel D with fixed-point beliefs. We thereby consider the low var(k) case in which the Dirichlet distribution has the concentration parameters $\alpha_i = 12$, and the high var(k) case in which $\alpha_i = 3$. The slopes of the optimal linear beliefs are chosen to maximize the utility of a CRRA investor with $\gamma = 2$, and they are reported in the row λ^{-1}. Profits are likewise the expected portfolio returns of $\gamma = 2$ investor, and certainty equivalents 'cer(γ)' are calculated for $\gamma = 2, 4, 10$, with portfolio weights calculated as in Eq. (6). Momentum returns are given by $m + \epsilon$ where ϵ is randomly drawn from a zero-mean normal distribution with standard deviation 0.125. Cer(γ) is an arithmetic return, and all other statistics are based on log returns.

	Panel A		Panel B		Panel C		Panel D	
var(k)	low	high	low	high	low	high	low	high
λ^{-1}			1.50	1.50	1.38	1.12		
Expected momentum returns m								
mean	3.0%	3.0%	2.7%	-2.4%	3.4%	4.2%	3.0%	3.0%
stdev	0.8%	1.4%	19.5%	174.2%	1.4%	2.0%	1.1%	1.6%
skew	0.5	0.6	-300.5	-151.3	-0.3	-0.3	0.3	0.4
kurt	3.3	3.1	92991.2	29218.7	4.7	10.8	3.2	3.0
min	0.74%	0.05%	-6046.58%	-38957.17%	-15.64%	-53.10%	-1.38%	-2.55%
max	7.57%	10.26%	10.38%	13.16%	10.78%	13.28%	9.36%	11.53%
Realized momentum returns $m + \epsilon$								
profit	3.17%	3.65%	-58.05%	-4863.08%	2.18%	0.65%	2.98%	3.44%
cer(2)	2.37%	2.62%	-100.00%	-100.00%	1.94%	0.74%	2.28%	2.53%
cer(4)	1.18%	1.30%	-100.00%	-100.00%	0.96%	0.37%	1.13%	1.25%
cer(10)	0.47%	0.52%	-100.00%	-100.00%	0.38%	0.15%	0.45%	0.50%

Table 2: Descriptive statistics of momentum returns

In Panels A and B the indicated variable is computed by institution (i.e., 13F filer) and then summarized across institutions. Qtrs, med., stdev., and mgd. refer to quarters, median, standard deviation, and managed, respectively. Assets are in units of $100 million and turnover is quarterly. Momentum investors refer to institutions classified as a momentum trader by one of our measures for at least 2/3 of the available quarters. Crowd refers to Cnt_1qrt, Cnt_1qrtP1, and Cap_1qrt; likewise the _4qrt extensions (as defined in Section 5.1). $\hat{\sigma}_{\text{Crowd}}$ is the estimate of volatility from a GARCH(1,1) model. Panel C contains factor exposures of quarterly momentum returns on the Fama-French three factor model and a dynamic extension with the three factors interacted with dummies for positive past annual factor returns. Alphas are monthly and t-statistics use White standard errors.

Panel A. Institutions

	all			momentum investors			not momentum investors		
	mean	med.	stdev.	mean	med.	stdev.	mean	med.	stdev.
#Qtrs of data	34.3	24.0	32.1	34.3	24.0	32.1	34.3	24.0	32.1
#Qtrs missing	3.6	0.0	9.5	3.6	0.0	9.5	3.6	0.0	9.5
#Stocks held	143.2	62.9	275.5	213	90.1	372.5	123.1	55.2	236.6
Assets mgd.	15.2	2.0	102.4	24.6	2.1	161.6	12.3	1.9	76.6
Turnover	0.21	0.16	0.17	0.24	0.18	0.18	0.21	0.15	0.16
#Institutions	6,360			1,414			5,059		

Panel B. Crowding variables

	Cnt_				CntP1_				Cap_			
	mean	med.	stdev.	ar(1)	mean	med.	stdev.	ar(1)	mean	med.	stdev.	ar(1)
_1qtr measure:												
ΔCrowd	0.000	0.004	0.075	-0.579	0.000	0.002	0.077	-0.563	0.000	0.001	0.034	-0.092
Crowd	0.492	0.495	0.055	0.083	0.504	0.509	0.054	0.001	0.050	0.042	0.051	0.782
$\hat{\sigma}_{\text{Crowd}}$	0.052	0.049	0.010	0.367	0.053	0.052	0.005	0.523	0.029	0.023	0.018	0.703
_4qtr measure:												
ΔCrowd	-0.001	0.000	0.017	-0.153	-0.001	0.000	0.017	-0.181	0.000	0.001	0.012	0.029
Crowd	0.117	0.120	0.029	0.818	0.102	0.101	0.026	0.770	0.019	0.014	0.020	0.810
$\hat{\sigma}_{\text{Crowd}}$	0.017	0.016	0.003	0.881	0.018	0.017	0.003	0.756	0.011	0.008	0.007	0.666

Panel C. Returns

	FF3					dynamic FF3							
	alpha	mkt	SMB	HML	R^2	alpha	mkt	SMB	HML	Dmkt	DSMB	DHML	R^2
coefficient	0.016	-0.35	-0.48	-0.59	12%	0.014	-0.85	-0.68	-0.95	0.81	0.48	1.02	25%
t-statistic	(4.4)	(-2.0)	(-2.1)	(-2.1)		(4.3)	(-2.9)	(-2.7)	(-2.7)	(2.8)	(1.2)	(2.1)	

Table 3: Transition frequencies

The table presents the probability of transitioning from the event in the row heading to that in the column heading at the indicated time (q indexes quarters), conditional on the later period not containing a missing observation. Panel A tabulates the transition at the level of individual institutions in terms of the momentum classification used to construct the indicated crowding variable ('1' stands for being a momentum trader and '0' for not being a momentum trader). Panel B tabulates stocks' membership in the winner, loser, or middle deciles of the momentum ranking. 'All' refers to the unconditional probability of classification.

Panel A. Institutions' trading

		q+1		q+4		
Indicator for > 0:		1	0	1	0	All
Cnt_/Cap_1qrt	1	0.56	0.44	0.55	0.45	0.48
	0	0.41	0.59	0.41	0.59	0.52
Cnt_/Cap_4qrt	1	0.69	0.31	0.29	0.71	0.11
	0	0.04	0.96	0.09	0.91	0.89
Cnt_1qrtP1	1	0.55	0.45	0.54	0.46	0.50
	0	0.44	0.56	0.45	0.55	0.50
Cnt_4qrtP1	1	0.64	0.36	0.19	0.81	0.10
	0	0.04	0.96	0.08	0.92	0.90

Panel B. Stock returns

	q+1			q+4			
	Win.	mid	Los.	Win.	mid	Los.	All
Winner	0.55	0.42	0.02	0.16	0.60	0.23	0.13
mid	0.08	0.83	0.09	0.12	0.74	0.14	0.68
Loser	0.02	0.34	0.64	0.17	0.52	0.31	0.18

Table 4: Momentum factor returns on crowding measures

Each column represents a predictive regression of quarterly momentum factor returns (1981 - 2015) on crowding. Each panel presents three specification: (1) without controlling for risk-factors; (2) controlling for the Fama and French three factor model; and (3) controlling for the dynamic factor model with the three factors interacted with dummies for positive past annual factor returns. Each set considers the three indicated crowding measures. The regressor 'Crowd' refers to the level of the crowding measure at the end of quarter q-1; $\Delta\text{Crowd}_\text{q}$ refers to the change over quarter q; and $\hat{\sigma}_\text{Crowd}$ is the estimate of volatility from a GARCH(1,1) model. 'Realized vol. of Mom rets.' is a control variable equal to the lagged realized volatility of daily momentum returns over the previous quarter (intercepts not tabulated). The t-statistics are computed with White standard errors.

Panel A. Crowding measures constructed using four-quarter trading histories									
Model:	cumulative returns			FF3			dynamic FF3		
Measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
$\Delta\text{Crowd}_\text{q}$	-0.57	-0.59	-0.27	-0.57	-0.59	-0.32	-0.55	-0.62	-0.28
	(-2.8)	(-2.8)	(-1.0)	(-3.2)	(-3.6)	(-1.3)	(-2.8)	(-3.0)	(-0.9)
$\text{Crowd}_\text{q-1}$	-0.17	-0.04	0.40	-0.14	0.10	0.41	-0.11	0.09	0.48
	(-1.1)	(-0.3)	(1.8)	(-1.2)	(0.7)	(2.1)	(-1.0)	(0.7)	(2.5)
$\hat{\sigma}_\text{Crowd}$	2.39	2.76	-0.20	2.53	3.26	-0.04	1.81	2.42	-0.36
	(1.4)	(1.6)	(-0.3)	(1.7)	(2.4)	(-0.1)	(1.3)	(1.6)	(-0.6)
Realized vol.	-0.36	-0.32	-0.31	-0.36	-0.29	-0.32	-0.30	-0.23	-0.26
of Mom rets.	(-1.9)	(-1.8)	(-1.6)	(-2.6)	(-2.1)	(-2.4)	(-2.6)	(-1.9)	(-2.3)
Adj-rsquare	13.5%	14.4%	10.5%	26.6%	28.2%	24.4%	35.3%	37.6%	35.0%

Panel B. Crowding measures constructed using one-quarter trading histories									
Model:	cumulative returns			FF3			dynamic FF3		
Measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
$\Delta\text{Crowd}_\text{q}$	-0.07	-0.06	-0.06	-0.07	-0.07	-0.11	-0.12	-0.09	-0.13
	(-1.0)	(-0.7)	(-0.7)	(-1.1)	(-0.9)	(-1.3)	(-1.3)	(-1.1)	(-1.2)
$\text{Crowd}_\text{q-1}$	-0.04	-0.04	0.14	-0.03	-0.01	0.12	0.02	0.00	0.17
	(-0.4)	(-0.4)	(1.4)	(-0.4)	(-0.1)	(1.5)	(0.2)	(-0.0)	(2.3)
$\hat{\sigma}_\text{Crowd}$	0.15	0.68	0.05	0.26	1.01	0.14	0.29	0.89	-0.02
	(0.5)	(0.9)	(0.2)	(1.0)	(1.7)	(0.7)	(0.7)	(1.1)	(-0.1)
Realized vol.	-0.29	-0.30	-0.32	-0.30	-0.32	-0.34	-0.23	-0.26	-0.28
of Mom rets.	(-1.6)	(-1.6)	(-1.6)	(-2.1)	(-2.1)	(-2.4)	(-1.8)	(-2.0)	(-2.4)
Adj-rsquare	6.9%	7.2%	9.9%	20.2%	21.0%	23.8%	33.8%	32.4%	35.9%

Table 5: Crowding and the left-tail of momentum returns

Each column represents a Probit regression with an indicator for next-quarter momentum returns in the bottom 10% (Panel A) or 5% (Panel B) of the full-sample (1981 - 2015) distribution. Each panel presents two sets of dependent variables: (1) the 3 month return on the momentum portfolio; and (2) its residual on the dynamic Fama and French model with the three factors interacted with dummies for positive past annual factor returns. Each set considers the three indicated crowding measures. In all cases the crowding measure is constructed using a four-quarter trading history. The regressor 'Crowd' refers to the level of the crowding measure at the end of quarter q-1; ΔCrowd_q refers to the change over quarter q; and $\hat{\sigma}_{\text{Crowd}}$ is the estimate of volatility from a GARCH(1,1) model. 'Realized vol. of Mom rets.' is a control variable equal to the lagged realized volatility of daily momentum returns over the previous quarter (intercepts not tabulated). T-statistics for the coefficient estimates are reported in parenthesis. Wald test p-values are reported in square brackets, testing the null hypothesis that a regressors effect on the left tail probability is equal in magnitude and opposite in sign to the (untabulated) effect on the right tail probability.

	Panel A. Predicting the 10% left tail						Panel B. Predicting the 5% left tail					
Dependent variable:	cumulative returns			dynamic FF3 residuals			cumulative returns			dynamic FF3 residuals		
Crowding measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
ΔCrowd_q	25.6	26.9	11.7	25.1	51.3	16.4	32.1	59.9	52.9	1.6	28.6	34.9
	(2.5)	(2.0)	(0.8)	(2.4)	(3.2)	(1.0)	(2.2)	(2.6)	(2.1)	(0.1)	(1.7)	(1.4)
	[0.08]	[0.28]	[0.87]	[0.69]	[0.04]	[0.79]	[0.31]	[0.06]	[0.07]	[0.24]	[0.65]	[0.32]
$\text{Crowd}_{q\text{-}1}$	11.6	10.0	-7.0	9.6	6.5	-7.8	20.2	11.7	-18.2	7.9	10.2	-21.6
	(1.8)	(1.3)	(-0.8)	(1.5)	(0.7)	(-0.9)	(1.8)	(0.9)	(-1.4)	(1.0)	(0.9)	(-1.5)
	[0.21]	[0.18]	[0.76]	[0.26]	[0.11]	[0.42]	[0.20]	[0.58]	[0.32]	[0.62]	[0.27]	[0.58]
$\hat{\sigma}_{\text{Crowd}}$	20.3	14.8	25.1	54.4	16.5	30.4	80.3	-15.0	3.3	76.0	67.8	31.4
	(0.4)	(0.3)	(1.2)	(1.2)	(0.3)	(1.4)	(1.2)	(-0.2)	(0.1)	(1.5)	(1.1)	(1.6)
	[0.04]	[0.02]	[0.96]	[0.05]	[0.13]	[0.55]	[0.02]	[0.67]	[0.65]	[0.02]	[0.20]	[0.00]
Realized vol.	14.0	13.1	10.6	12.2	11.7	10.5	17.2	14.9	17.1	10.6	10.2	11.4
of Mom rets.	(4.3)	(4.0)	(3.0)	(3.8)	(3.2)	(3.4)	(3.7)	(3.2)	(3.4)	(3.0)	(2.8)	(3.0)
	[0.00]	[0.00]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Table 6: Conditional volatility, skewness and kurtosis of momentum returns

We split the sample of monthly momentum returns (1981 - 2015) into terciles according to crowding ('Crowd'), change in crowding ('ΔCrowd') or volatility in the last available quarter for each month. The G1 stands for the bottom tercile, G2 for the second tercile and G3 for the top tercile. The values in parenthesis are t-statistics for the difference between G3 and G1 obtained with the delta method.

	ΔCrowd			Crowd			Realized vol.
	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	of Mom rets.
Volatility							
G1	27.3	25.9	30.0	33.5	33.0	21.8	15.3
G2	27.4	26.9	21.0	19.4	21.3	30.6	17.3
G3	23.2	25.2	26.5	23.2	22.1	24.8	38.7
	(-1.0)	(-0.2)	(-0.7)	(-2.0)	(-2.1)	(1.1)	(5.7)
Skewness							
G1	-1.83	-1.37	-1.34	-2.05	-2.13	-0.48	-0.34
G2	-1.94	-2.75	-0.66	0.34	-0.07	-2.54	-0.31
G3	-0.22	-0.17	-1.97	-0.18	0.16	0.07	-1.18
	(1.8)	(1.3)	(-0.7)	(3.3)	(4.1)	(1.1)	(-2.0)
Kurtosis							
G1	13.97	9.99	10.30	10.44	10.65	4.75	4.03
G2	11.79	16.47	5.81	4.16	4.46	13.65	4.06
G3	4.48	5.59	12.68	5.35	5.80	5.22	6.52
	(-4.3)	(-1.3)	(0.7)	(-2.6)	(-2.4)	(0.4)	(2.1)

Table 7: Volatility in momentum factor returns on crowding measures

Each column represents a predictive regression of realized volatility in daily momentum factor returns over the next quarter (1981 - 2015) on crowding. Each panel presents three sets of dependent variables using daily: (1) raw returns on the momentum portfolio; (2) residual returns using the Fama and French three factor (FF3) model; and (3) residual on FF3 using dynamic weights. Each set considers the three indicated crowding measures. The regressor 'Crowd' refers to the level of the crowding measure at the end of quarter q-1; ΔCrowd_q refers to the change over quarter q; and $\hat{\sigma}_{\text{Crowd}}$ is the estimate of volatility from a GARCH(1,1) model. 'Realized vol. of Mom rets.' is a control variable equal to the lagged realized volatility of daily momentum returns/residuals over the previous quarter (intercepts not tabulated). The t-statistics are computed with Newey-West standard errors with 3 lags.

Panel A. Crowding measures constructed using four-quarter trading histories									
Dependent variable:	vol of returns			vol of FF3 residuals			vol of dynamic FF3 residuals		
Crowding measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
ΔCrowd_q	-0.19	-0.45	-0.06	-0.21	-0.44	-0.12	-0.19	-0.38	-0.17
	(-1.1)	(-3.2)	(-0.2)	(-1.2)	(-3.1)	(-0.4)	(-1.0)	(-2.4)	(-0.6)
$\text{Crowd}_{q\text{-}1}$	-0.14	-0.17	0.05	-0.16	-0.17	0.00	-0.12	-0.15	0.05
	(-2.3)	(-1.9)	(0.4)	(-2.5)	(-2.0)	(0.0)	(-2.2)	(-2.4)	(0.5)
$\hat{\sigma}_{\text{Crowd}}$	0.65	1.02	0.68	0.96	1.34	0.86	0.45	0.44	0.51
	(0.8)	(1.2)	(1.9)	(1.1)	(1.5)	(2.3)	(0.8)	(0.7)	(2.0)
Realized vol.	0.76	0.77	0.76	0.72	0.73	0.72	0.73	0.74	0.72
of Mom rets.	(7.4)	(8.1)	(8.1)	(7.1)	(7.9)	(8.2)	(6.7)	(7.5)	(7.6)
Adj-rsquare	63.7%	66.2%	64.2%	60.7%	63.4%	61.7%	59.7%	62.5%	60.7%
Panel B. Crowding measures constructed using one-quarter trading histories									
Dependent variable:	vol of returns			vol of FF3 residuals			vol of dynamic FF3 residuals		
Crowding measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
ΔCrowd_q	-0.11	-0.08	-0.02	-0.10	-0.08	-0.04	-0.10	-0.08	-0.06
	(-2.8)	(-1.8)	(-0.1)	(-2.9)	(-2.0)	(-0.4)	(-3.4)	(-2.3)	(-0.7)
$\text{Crowd}_{q\text{-}1}$	-0.07	-0.02	0.04	-0.06	-0.01	0.03	-0.05	-0.03	0.04
	(-1.5)	(-0.3)	(1.3)	(-1.6)	(-0.2)	(0.8)	(-1.6)	(-0.6)	(1.5)
$\hat{\sigma}_{\text{Crowd}}$	0.50	1.38	0.10	0.60	1.55	0.16	0.39	1.02	0.08
	(2.3)	(2.6)	(0.6)	(2.7)	(2.9)	(0.9)	(1.9)	(2.1)	(0.7)
Realized vol.	0.78	0.77	0.78	0.75	0.74	0.74	0.75	0.74	0.75
of Mom rets.	(8.3)	(7.8)	(7.3)	(8.5)	(7.9)	(7.1)	(7.6)	(7.1)	(6.9)
Adj-rsquare	66.0%	66.0%	63.2%	63.5%	63.6%	60.0%	63.5%	62.8%	60.0%

Table 8: Momentum factor returns as a determinant of crowding

Each column represents a predictive regression of a different crowding measure on lag momentum returns and lag momentum realized volatility. Panel A shows the results for four-quarter measures and Panel B for one-quarter measures. Volatility is computed using daily momentum returns (intercepts not tabulated). The t-statistics are computed with Newey-West standard errors with 3 lags.

	Panel A			Panel B		
Crowding horizon:	4qtr			1qtr		
Crowding measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
1yr return$_{q\text{-}1}$	0.23	0.06	0.27	0.47	0.40	0.73
	(1.2)	(0.5)	(2.3)	(2.0)	(2.2)	(2.3)
1yr return$_{q\text{-}5}$	0.52	0.48	0.16	0.66	0.68	0.31
	(2.9)	(3.7)	(1.4)	(2.6)	(3.3)	(1.0)
1yr volatility$_{q\text{-}1}$	-0.41	-0.31	-0.05	-0.46	-0.37	-0.10
	(-3.3)	(-2.9)	(-0.9)	(-2.9)	(-3.0)	(-0.7)
1yr volatility$_{q\text{-}5}$	0.11	0.04	-0.07	0.18	0.07	-0.24
	(0.9)	(0.4)	(-0.9)	(1.3)	(0.5)	(-1.2)
Adj-rsquare	20.0%	17.5%	16.3%	6.5%	6.7%	18.9%

Table 9: Regressions of momentum return moments on crowd count and crowd capital jointly estimated

Each column presents a regression of the indicated momentum return metric as the dependent variable and the indicated horizon for estimating the crowding measure (1qtr or 4qtr). In the case of 'left-tail' the regression is Probit as in Table 5. Return refers to the dynamic Fama-French 3 factor residual for the Probit and Volatility panels, and to a regression with the dynamic FF3 factors as controls in the Returns panel. All regressors in the row headings are included in each regression. Thus, the regressions in the columns labelled 'Returns' correspond to Table 4 using Cnt_ and Cap_ and DFF3 (but estimated jointly). Likewise, the regressions in the columns labelled 'left-tail' and 'Volatility' correspond to jointly estimated versions of Tables 5 and 7, respectively, for the case of Cnt_ and DFF3. T-statistics are reported in parenthesis, and the p-values of a Wald test that the effect of a regressor on the left and corresponding right tail (not tabulated) sum to zero are reported for the probits in brackets.

Dependent variable:	Returns		10% left-tail		5% left-tail		Volatility	
Crowding horizon:	1qtr	4qtr	1qtr	4qtr	1qtr	4qtr	1qtr	4qtr
Crowd = _Cnt:								
ΔCrowd$_q$	-0.11	-0.51	6.7	26.2	10.3	-1.0	-0.12	-0.13
	(-1.7)	(-3.1)	(1.8)	(2.4)	(1.5)	(-0.1)	(-3.2)	(-1.2)
			[0.33]	[0.44]	[0.38]	[0.40]		
Crowd$_{q\text{-}1}$	-0.04	-0.32	6.5	14.5	16.7	19.9	-0.07	-0.15
	(-0.5)	(-2.7)	(1.3)	(1.9)	(2.2)	(1.8)	(-2.0)	(-2.7)
			[0.28]	[0.63]	[0.14]	[0.96]		
$\hat{\sigma}_{\text{Crowd}}$	-0.03	3.03	-2.9	7.8	43.1	-10.2	0.39	0.24
	(-0.1)	(2.2)	(-0.1)	(0.1)	(1.4)	(-0.1)	(2.3)	(0.4)
			[0.94]	[0.19]	[0.60]	[0.26]		
Crowd = _Cap:								
ΔCrowd$_q$	-0.03	0.07	3.7	9.5	21.6	24.9	0.07	-0.07
	(-0.3)	(0.3)	(0.6)	(0.6)	(2.0)	(0.9)	(0.9)	(-0.3)
			[0.73]	[0.75]	[0.07]	[0.29]		
Crowd$_{q\text{-}1}$	0.16	0.74	-3.9	-14.5	-12.5	-29.3	0.05	0.14
	(2.2)	(3.7)	(-1.1)	(-1.3)	(-1.7)	(-1.9)	(1.9)	(1.6)
			[0.65]	[0.46]	[0.38]	[0.83]		
$\hat{\sigma}_{\text{Crowd}}$	0.06	-0.96	10.3	36.5	-1.2	52.8	0.02	0.41
	(0.2)	(-1.4)	(1.1)	(1.5)	(-0.1)	(1.9)	(0.2)	(1.7)
			[0.42]	[0.92]	[0.49]	[0.09]		
Control:								
Realized vol.	-0.27	-0.36	11.1	14.1	16.6	15.7	0.75	0.68
of Mom rets.	(-2.2)	(-3.3)	(3.3)	(3.6)	(3.4)	(3.2)	(6.9)	(6.2)
			[0.00]	[0.01]	[0.00]	[0.06]		

Table 10: Robustness: conditional volatility, skewness, and kurtosis of momentum returns

To calculate each column we split monthly momentum returns (1981 - 2015) into terciles every four quarters according to the level of Huang (2015)'s momentum gap variable in the column 'Mom Gap', and the momentum gap variable orthogonal to the variables shown for the other columns. Variables not previously used are 'ΔMom Inst' which is the percentage difference in aggregate institutional ownership between past winners and losers (see, Huang, 2015), and 'Win Inst' which is the aggregate institutional ownership of the winner decile (see, Lou and Polk, 2013). G1 stands for the bottom tercile, G2 for the second tercile and G3 for the top tercile. The values in parenthesis are t-statistics for the difference between G3 and G1 obtained with the delta method.

	Mom Gap	orthogonal to					
		ΔMom Inst	Win Inst	Crowd Cnt_	Crowd CntP1_	Crowd Cap_	Realized vol. of Mom rets.
Volatility							
G1	12.8	12.2	13.5	14.3	14.5	12.9	21.0
G2	19.2	19.9	18.7	18.1	18.0	19.5	17.8
G3	38.6	38.5	38.6	38.6	38.6	38.4	35.6
	(6.4)	(6.7)	(6.2)	(6.0)	(5.9)	(6.7)	(3.2)
Skewness							
G1	-0.32	-0.37	-0.27	-0.40	-0.56	-0.41	-0.68
G2	-0.01	0.04	0.04	0.07	0.13	0.25	-0.22
G3	-1.30	-1.25	-1.30	-1.30	-1.31	-1.27	-1.51
	(-2.4)	(-2.2)	(-2.6)	(-1.9)	(-1.7)	(-2.1)	(-1.4)
Kurtosis							
G1	3.32	3.41	3.17	4.21	4.11	3.50	6.42
G2	3.84	3.66	4.19	4.07	4.04	4.60	4.83
G3	6.61	6.57	6.62	6.62	6.66	6.51	8.42
	(2.9)	(2.8)	(3.1)	(1.8)	(2.0)	(2.6)	(1.1)

Internet Appendix to accompany the paper

"Institutional Crowding and the Moments of Momentum"

(Not for publication)

This Internet Appendix contains two robustness checks for the simulation analysis in Section 3 of the paper. Our benchmark is the simulation with Dirichlet concentration parameters $\alpha_i = 12$, i.e., the low $var(k)$ case, and we investigate the impact of changing the distributional assumptions for δ and higher concentration parameters.

First, we ask whether our results are robust to using a uniform distribution for δ instead of a lognormal distribution. In particular, we let δ follow a uniform distribution on [0.06, 0.12], and leave the setting otherwise identical to the one in the paper. The results are reported in Figure IA.1 and Table IA.1. In summary, the results are very similar to those in Section 3's low $var(k)$ case. In the naïve beliefs case, ex-ante momentum returns again have pronounced negative skewness, high volatility and large excess kurtosis, and they are well behaved with low volatility, slightly positive skewness, and no excess kurtosis in the fixed-point beliefs case.

Second, we ask whether the beliefs specifications for unknown capital become more similar to the known capital case when $var(k)$ is very small. To achieve this, we set the concentration parameters $\alpha_i = 60$ in the Dirichlet distribution, and leave the setting otherwise identical to the one in the paper. The results in Figure IA.2 and Table IA.2 verify that crashes disappear in the naïve beliefs case once capital uncertainty is negligible. Ex-ante momentum returns in all four specifications are now well behaved and have similar return characteristics.

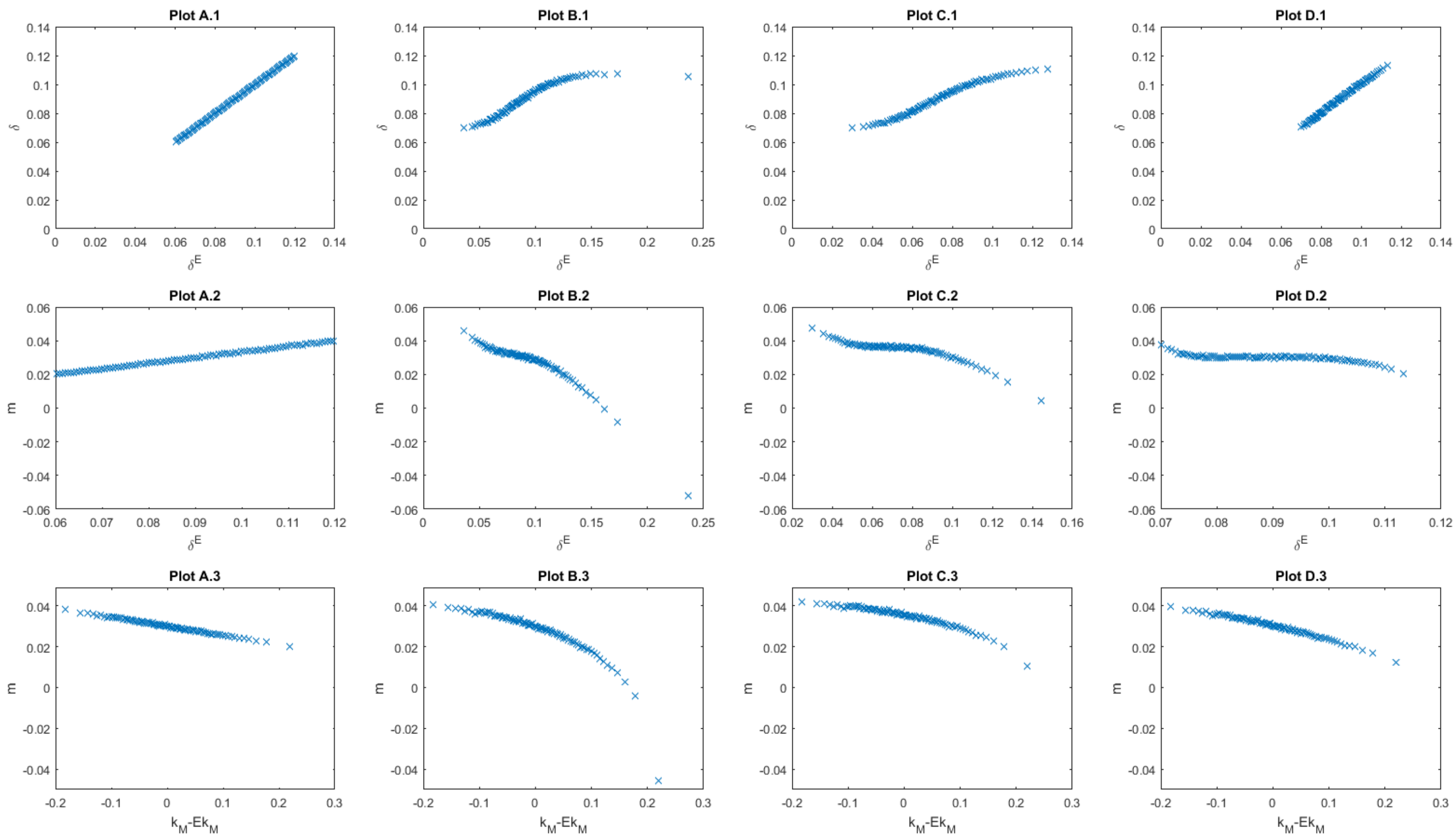


Figure IA.1: Beliefs and expected momentum returns in simulations – uniform distribution

This figure is constructed in the same way as Figure 1 in the paper. In particular, the simulations use 100,000 independent random draws of $\{k_I, k_M, \delta\}$, where k_I and k_M are informed and momentum capital, respectively, and δ is the signal of differential fundamental value for winners minus losers. k_I and k_M (and k_C) follow a Dirichlet distribution with concentration parameters $\alpha_i = 12$ (blue Xs), and $Ek_I = Ek_M = 1/3$. δ follows a uniform distribution on [0.06 0.12]. The market clearing formation period return f is solved for each $\{k_I, k_M, \delta\}$ pair by iteration using different specifications for momentum traders' beliefs δ^E: linear beliefs with known capital shares in Plots A.1-3, naïve linear beliefs in Plots B.1-3, optimal linear beliefs in Plots C.1-3, and fixed-point beliefs in Plots D.1-3. The expected momentum return is then $m = \delta - f$. The triplets $\{\delta, m, k_M\}$ are ranked into 100 equally populated bins according to the horizontal-axis variable, and the plots represent the averages for the indicated variables within these bins.

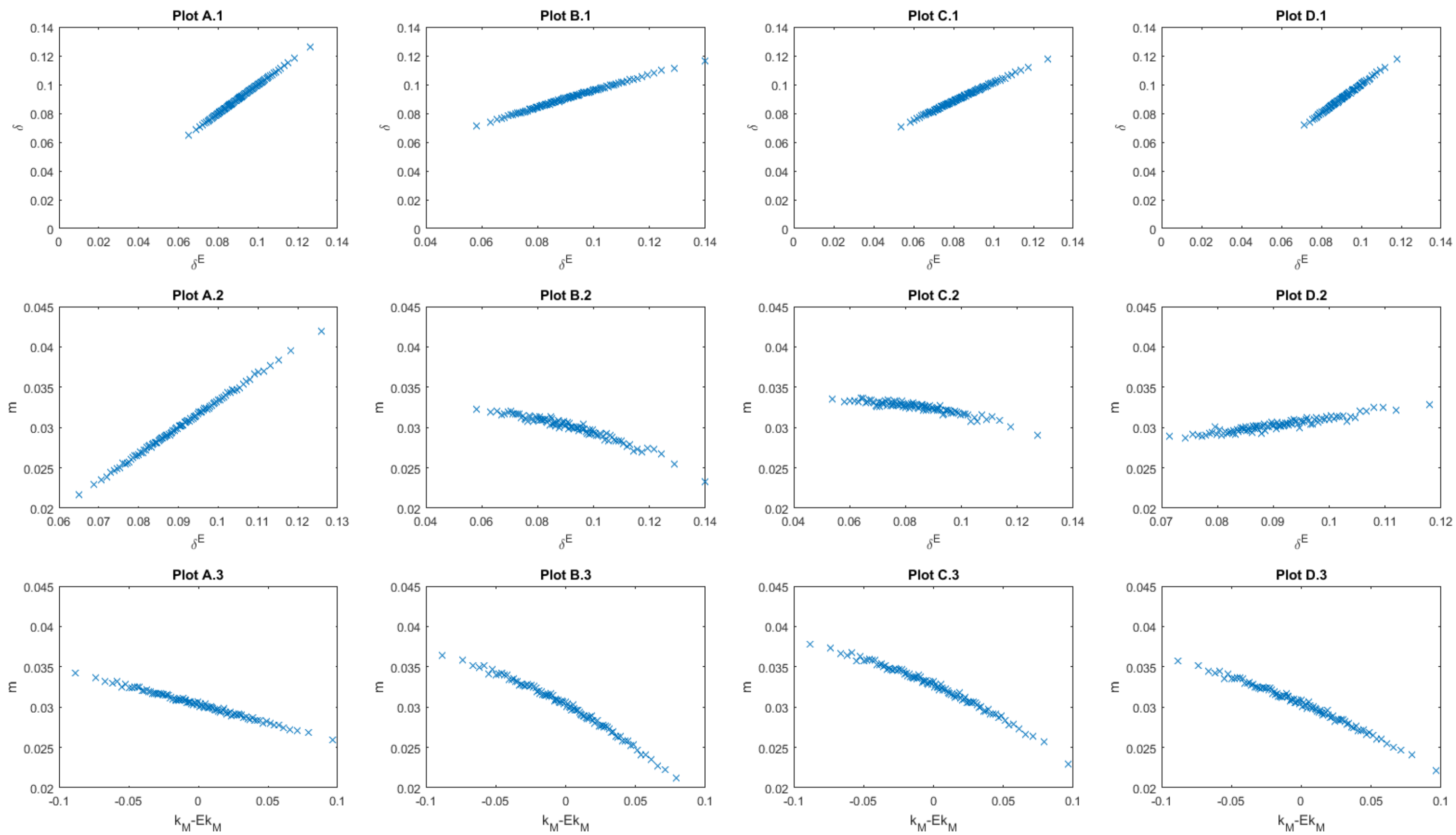


Figure IA.2: Beliefs and expected momentum returns in simulations – very low $var(k)$

This figure is constructed in the same way as Figure 1 in the paper, and the simulations are identical to those in Figure 1 except that k_I and k_M (and k_C) follow a Dirichlet distribution with concentration parameters $\alpha_i = 60$ (blue Xs), which still implies $Ek_I = Ek_M = 1/3$. In addition, as opposed to Figure IA.1 δ follows a lognormal distribution with $\mu = -2.405$ and $\sigma = 0.125$ implying an average δ of 9.1% with standard deviation of 1.14% as in the paper.

Table IA.1: Momentum returns in simulations – uniform distribution

The table reports unconditional return statistics for the simulations described in the caption of Figure IA.1. The top part contains the descriptive statistics of expected momentum returns across all simulations. Mean, stdev, skew, kurt, min and max refer to average, standard deviation, skewness, kurtosis, minimum, and maximum, respectively. Panel A reports the results for the linear beliefs and known capital case, Panel B is the case with naïve beliefs, Panel C with optimal linear beliefs, and Panel D with fixed-point beliefs. The Dirichlet distribution has the concentration parameters $\alpha_i = 12$, and δ follows a uniform distribution on $[0.06,\ 0.12]$. The slopes of the optimal linear beliefs are chosen to maximize the utility of a CRRA investor with $\gamma = 2$, and they are reported in the row λ^{-1}. Profits are likewise the expected portfolio returns of $\gamma = 2$ investor, and certainty equivalents 'cer(γ)' are calculated for $\gamma = 2, 4, 10$, with portfolio weights calculated as in (6). Momentum returns are given by $m + \epsilon$ where ϵ is randomly drawn from a zero-mean normal distribution with standard deviation 0.125. Cer(γ) is an arithmetic return, and all other statistics are based on log returns.

	Panel A	Panel B	Panel C	Panel D
λ^{-1}		1.50	1.34	
Expected momentum returns m				
mean	3.0%	2.8%	3.5%	3.0%
stdev	0.9%	2.7%	1.4%	1.4%
skew	0.5	-92.9	-0.1	0.3
kurt	3.1	17386.3	9.6	2.9
min	0.61%	-534.41%	-35.57%	-2.04%
max	7.51%	9.22%	9.38%	8.56%
Realized momentum returns $m + \epsilon$				
profit	3.11%	1.75%	1.99%	2.78%
cer(2)	2.30%	-100.00%	1.85%	2.16%
cer(4)	1.14%	-100.00%	0.92%	1.07%
cer(10)	0.45%	-100.00%	0.37%	0.43%

Table IA.2: Momentum returns in simulations – very low $var(k)$

The table reports unconditional return statistics for the simulations described in the caption of Figure IA.2 and is constructed in the same fashion as Table IA.1.

	Panel A	Panel B	Panel C	Panel D
λ^{-1}		1.50	1.44	
Expected momentum returns m				
mean	3.0%	3.0%	3.3%	3.0%
stdev	0.5%	0.8%	0.7%	0.7%
skew	0.4	0.2	0.3	0.3
kurt	3.4	3.3	3.3	3.3
min	1.39%	-0.39%	0.41%	0.18%
max	6.13%	7.12%	7.35%	7.35%
Realized momentum returns $m + \epsilon$				
profit	3.02%	2.90%	2.64%	2.91%
cer(2)	2.29%	2.15%	2.24%	2.24%
cer(4)	1.14%	1.07%	1.11%	1.11%
cer(10)	0.45%	0.43%	0.44%	0.44%