Speculation, Hedging and Interest Rates

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ABSTRACT

We study the bond markets implications of a heterogeneous economy with low risk aversion agents who use different models to interpret economic shocks. We show analytically that the interaction of low risk aversion and differences in beliefs gives rise to speculative motives for trading that help rationalise several features of Treasury bond markets that traditional models find difficult to explain. Empirically, we test model predictions using a large dataset on beliefs about fundamentals and find that disagreement (*i*) lowers the unconditional risk-free rate and makes its dynamics pro-cyclical, (*ii*) raises the slope of the yield curve, and (*iii*) increase bond risk premia and makes its dynamics counter-cyclical.

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Daily trading volume in U.S. Treasury bonds is about ten times daily U.S. GDP: \$476 billions versus \$48 billions, respectively.[1] Regulators have regularly commented that this amount of trading is unlikely to be exclusively due to daily hedging demands. The potential importance of a speculative channel is discussed in an extensive theoretical literature studying principal-agents models in which agents (financial institutions, hedge funds, and proprietory traders) have limited liability and convex incentives. These models show the emergence of endogenous risk shifting incentives and risk-taking behaviors that deviates from pure hedging motives. Since a significant component of trading in bond markets is done by institutional investors acting as agents, it is therefore important to study the potential role of speculative motives for equilibrium asset prices.[2]

In this paper, we study the interaction between the hedging and belief-based speculative channels for the dynamics of bond prices in the context of a model that allows both channels to emerge in equilibrium. The term structure literature that focuses on the hedging channel as a source of equilibrium pricing implications for bond prices has highlighted several tensions that usually arise from this channel. The short-term rate is empirically observed to be pro-cyclical, thus models with large hedging motives (which are well suited to explain large equity risk premia) give rise to negative bond risk premia, contrary to what is observed in the data. Moreover, the assumption of large risk aversion gives rise to counterfactually high risk free rates and is hardly realistic in the context of economies with large institutional investors.

This paper addresses two central questions. First, what are the relative theoretical roles of the hedging and speculative channels in terms of the properties of bond markets? Second, what is the extent to which belief heterogeneity can help to explain some of the empirical bond pricing puzzles?

To address these two questions, we work within the context of a widely studied family of rational models in which agents agree to disagree about some characteristics of the dynamics of fundamentals.[3] Heterogeneity in agents' beliefs gives rise to trade. When agents are very risk averse ($\gamma >> 1$) their trading activity is dominated by hedging motives; while when they are risk

[1] Source: www.sifma.org/research/statistics.aspx

[2] After the 2008 Financial Crisis, the American Congress, the U.S. Federal Reserve, and regulators across the world took significant regulatory steps to address concerns that the nature of institutional incentives in the financial industry may lead to significant speculation affecting asset prices and systemic stability.

[3] Important contributions in this literature include David (2008), who develops a model to explain the equity risk premium; (Buraschi, Trojani, and Vedolin, 2014, 2013), Bhamra and Uppal (2011); Gallmeyer and Hollifield (2008), and for a survey Basak (2005)

tolerant ($\gamma < 1$), their trading activity is motivated by speculative reasons. We derive closed-form solutions for bond prices that allow us to quantify the marginal effects of these two motives. We find that when moving from single to multiple agent models several properties of asset prices change. Differences in beliefs affect the equilibrium pricing kernel, becoming a source of predictable variation in risk prices with empirical implications linking (*i*) the pro-cyclicality of short term interest rates, (*ii*) the slope of the term structure; (*iii*) yield volatilities, and (*iv*) counter-cyclical bond risk premia to observable proxies for macroeconomic disagreement. First, we calibrate the model to quantify the potential importance of the speculative channel to learn whether it is possible to match, *at the same time*, these four sets of moments restrictions. Then, we use survey data from a large panel of professional forecasters' beliefs about macroeconomic variables to empirically test the conditional implications of the model in terms of the joint dynamics of disagreement and bond markets. All empirical results are consistent with the implications of an economy with risk tolerant agents ($\gamma < 1$), which points to investor speculation being a primitive determinant of fixed income markets. The result is interesting given the increasing literature that argues about the important role played by intermediaries and hedge funds on asset prices. The result also passes the Occam test, as this channel does not require additional ingredients such as preferences for early resolution of uncertainty, exogenous stochastic volatility, or exogenous specifications for the price of inflation risk.[4]

These findings are important since a substantial empirical literature documents that homogeneous agent structural models, designed to understanding the properties of equity markets, have met substantial difficulties in explaining properties of bond markets. For example, in the long run risks model (Bansal and Yaron (2004)) positive shocks to expected consumption growth drive up real interest rates so the long-run risk models with positive risk prices for consumption growth (as required by equity) generates real bonds which are hedges (thus earning a negative risk premium). Indeed, Beeler and Campbell (2009) argue *'the behaviour of the real term structure is a troubling difficulty for the long-run risks model of asset prices'*.[5] Habit models also have difficulties when confronted with the real term structure. In particular, Duffee (2012) estimates over long sample

[4]Some important contributions following this approach include Wachter (2006), Buraschi and Jiltsov (2005), Piazzesi and Schneider (2007), Gallmeyer, Hollifield, Palomino, and Zin (2007), and Bansal and Shaliastovich (2013).

[5]Moreover, evidence from inflation protected markets show the slope of the real yield curve is (on average) flat or slightly positive and highly correlated with the slope of the nominal yield curve.

periods that (i) consumption shocks and their lags have almost no correlation with real short rates; and (ii) '*there is no statistically significant relation between the surplus and future excess bond returns*'. Indeed, if consumption surplus is high in recessions given the empirically observed pro-cyclicality of short term interest rates, long term bonds should be a hedge. More generally, a large empirical literature based on laboratory experiments also shows that agents have much lower effective risk aversion than commonly assumed in single agent consumption asset pricing models (the 'Mehra Prescott Puzzle').

To study the relative importance of these two channels, we begin by borrowing consumption dynamics from the long-run risk literature, where real growth rates contain a highly persistent, low volatility, time-varying conditional mean component. Hansen, Heaton, and Li (2008) argue that if long run risk exist, then agents within these models must face significant measurement challenges in quantifying the long-run risk-return trade-off.[6] We embed this observation within the model by assuming agents hold subjective models for these long-run risks.[7] This induces agents to demand different portfolios of state contingent claims; thus, generating trade in equilibrium. In these economies, each agent has his own stochastic discount factor and shocks to their individual beliefs cannot be fully insured, independently of the number of contingent claims. This is similar to the effect of labor income shocks in incomplete markets economies, with the difference that the asset pricing effects do not require frictions for these individual shocks to affect prices. This simple extension to a homogeneous CRRA economy generates time-variation in both *quantities* and *prices* of risk.

In this economy, subjective beliefs affect short term rates in two ways. The first effect is due to wealth weighted aggregation of agents' beliefs. This effect depends on the history of beliefs: past disagreement drives speculative trade which results in an endogenous redistribution of wealth over time toward the agent whose model aligned more closely with the data generating process (Friedman (1953b)'s survivorship effect). The second effect is due to optimal hedging demand and depends on whether agents are risk averse or risk tolerant. As agents rely on different

[6]Hansen, Heaton, and Li (2008) suggest that *'the same statistical challenges that plague econometricians presumably also plague market participants'.*

[7]Disagreement about the state of the economy is often discussed in the financial press. For example, to quote a recent article, 'Whether rates will be high or low a few years from now has very little to do with what the Fed does this week. It has quite a lot to do with what happens to forces deep inside the economy that are poorly understood and extremely hard to forecast.' The same article highlights the substantial disagreement among economists about long-term growth and interest rates. New York Times 15/12/2015.

models, shocks to fundamentals endogenously change the *perceived* investment opportunity set, affecting agents' future speculative trading opportunities. This occurs even if fundamentals are homoskedastic as long as agents agree to disagree. When agents are risk tolerant larger differences in beliefs reduce the short term interest rate. This is because the substitution effect dominates the wealth effect and agents reduce current consumption when the investment opportunity set is impacted by belief shocks. The opposite holds for risk averse agents (positive hedging demand). This is is interesting since it naturally introduces a channel that can help to address the risk-free rate channel.

A second implication relates to the shape of the yield curve. When the endowment process follows affine dynamics, bond yields are given by the sum of an exponentially affine function in the expected economic growth plus a quadratic function in disagreement. Our calibration exercise demonstrates how the distribution of wealth, risk aversion and disagreement interact to affect the shape of the yield curve. A robust feature is that when $\gamma \neq 1$, the yield curve can be upward sloping even when the equivalent economy with homogeneous investors would give rise to a decreasing yield curve. This is interesting given the well known difficulty for single agent economies to produce upward sloping term structures without the introduction of exogenous inflation dynamics.

The third implication is related to compensation for holding interest rate risk. We study bond risk premia under the measure of an unbiased econometrician and find properties differ from those arising in homogeneous economy in several respects. There are two channels that contribute and the link between risk aversion and risk premia is not monotone. The first one is standard and proportional to the product of risk aversion γ and the volatility of aggregate consumption. It is well known that this term requires large risk aversion to match the risk premium observed in the data. The second channel, on the other hand, is non-monotonic in γ. Indeed, for lower levels of risk aversion agents are willing to speculate more, so that the endogenous quantity of risk that each agent faces is larger. The net effect of this non linearity is that for γ below a threshold, the risk premium can be high even if γ is low. In this case, the economy is able to generate sizeable time-varying risk premia on bonds that are positive on-average and, at the same time, generate a low real risk free rate.

Finally, the model also predicts that bond prices depend on the history of beliefs. This is due to the fact that equilibrium prices depend on agents' wealth distribution, which is the result of past trade. Thus, distant lags of disagreement should be statistically significant to explain the dynamics of expected bond excess returns.

This rich set of testable implications makes bond markets an ideal testing ground to learn about the relative importance of the speculative versus hedging channel in heterogeneous agent economies.

Taking the model to the data, we construct a rich data set on the distribution of expectations from professional forecasters about real GDP growth and inflation. This dataset is unique in that it is available at a monthly frequency, covers a long history, and it is based on a large and stable cross-section of forecasters. Exploiting information in the cross-sectional distribution of expectations we extract an observable proxy for disagreement that allows us to study the testable implications from the model. Our empirical results are summarised as follows.

First, we find that both real and nominal short term interest rates are negatively related to current disagreement about fundamentals. In both cases disagreement about real economic growth is statistically significant with t-statistics of -3.40 and -3.58, respectively, after controlling for expected inflation and real economic growth consensus expectations. We check whether the result is due to persistence in interest rates and run a complementarity regression in yield changes (one year changes in the one year Treasury yield). We find that, even after controlling for the Fama and Bliss (1987) one-year forward spot spread and the Cochrane and Piazzesi (2005) factor, disagreement is negatively related to short term interest rates. The negative sign of the slope coefficient is consistent with models in which $\gamma < 1$.

Second, we find a strong link between real disagreement and the slope of the term structure. A one standard deviation shock to disagreement raises the slope of the yield curve by 0.20 standard deviations with a robust t-statistic of 3.49. The statistical significance is retained even after controlling for expected economic growth and inflation. Even in this case, the sign of the slope coefficient is consistent with models in which the speculative channel plays a significant role ($\gamma < 1$). We also find that when both real and inflation disagreement are included jointly, after controlling for expected inflation and growth, real disagreement is statistically significant but disagreement on inflation is no longer significant. This highlight the importance of a real channel of disagreement even in the context of nominal Treasury bonds.

Third, when we study a set of implications that focus on the conditional dynamics of excess bond returns, we find that a substantial amount of return predictability is arising from both contemporaneous and lagged dispersion terms. In the case of 2-year bonds, contemporaneous real disagreement is significant with a t-statistics of 2.91 and 3-months lagged disagreement is significant with a t-statistics of 3.37. We also find that for excess returns, most of the predictability comes from time variation in real disagreement as opposed to beliefs on inflation. When we investigate the specific role of past speculative activity, we find that lagged real disagreement is indeed very significant in both economic and statistical terms. For 5-year bonds, lagged disagreement is the dominant component of predictability. This result is robust to the use of different lags of real disagreement up to 6 months. This is an intriguing result that suggests that past speculative activity, as proxied by lagged disagreement, has a large effect on the endogenous characteristics of the marginal investor.

Considering the theoretical predictions arising from the model, together with the results from the three empirical tests, one common learning point emerges: many of the stylised facts of bond markets can be understood in terms of economies with heterogeneous speculative ($\gamma < 1$) agents trading on their subjective beliefs. These results are consistent with a large literature studying endogenous incentives in economies with financial intermediation in which financial institutions (such as hedge funds, pension funds, mutual funds, and other financial intermediaries) with low risk aversion play a significant role on equilibrium outcomes and pricing of state continent claims. Indeed, a large literature studies how convex performance-based incentives and contracts limiting managers' liability as a consequence of the their decisions can induce risk-shifting and even risk-seeking behaviours.

Related Literature

This paper is related to three streams of the asset pricing literature. A first stream documents the difficulty to reconcile the properties of the yield curve with homogeneous agent macro models (Duffee (2012)). Several studies show strong evidence of bond return predictability. While it is possible to specify structural models that generate predictability this often comes at the cost of missing other important properties of the yield curve. For example, Beeler and Campbell (2009) argue that while long-run risks models can give rise to significant bond risk premia, they can produce either pro-cyclical real short term interest rates or upward sloping real term structure, but not both at the same time. Indeed, a necessary condition for an upward sloping term structure is negative autocorrelation in consumption growth. However, consumption growth shows positive autocorrelation which, in these models, would imply a negative average risk premium for real bonds.[8] To overcome this tension, a stream of the term structure literature have investigated models that combine an exogenous inflation dynamics with recursive or non-linear preferences.[9] With respect to this literature, we take a different direction. Instead of introducing more general preferences or technology, we simply introduce an additional agent but allow him to engage in belief-based trade in equilibrium. This feature gives rise to endogenous time-variation in both quantity and price of risk, which we find helps to explain several bond puzzles without generating excess interest rate volatility.

This paper also extends a literature that studies economies with heterogeneous beliefs and trade in equilibrium. Sources of heterogeneity supporting trade include preferences, wealth and beliefs. In this paper, we focus on the last dimension and investigate the empirical implications of heterogeneity in macroeconomic beliefs for bond returns. Contributions to this literature include Basak (2005) who studies the impact of nonfundamental risk on asset prices, Buraschi and Jiltsov (2006) who examine disagreement and option pricing, David (2008) and Dumas, Kurshev, and Uppal (2009) who focus on equity markets, and Chen, Joslin, and Tran (2012) who investigate the pricing of rare disaster risk. Buraschi and Jiltsov (2006) is the first to combine heterogeneity in beliefs and intertemporal learning, while David (2008) is the first to highlight the importance of small risk aversion in belief-based trading models. None of these contributions, however, discuss bond market implications.[10] In the context of bond markets, Xiong and Yan (2010) show that when agents are myopic bond prices can deviate from those implied by the average consensus beliefs. The beliefs of the representative agent include an aggregation bias that depends on the relative wealth distribution of agents. This particular feature of heterogeneous agent aggregation is also discussed by Jouini, Marin, and Napp (2010). In the context of an economy where the exogenous price level

[8]See, for example, Bansal and Yaron (2004) who report a first order autocorrelation in time-averaged growth rates of real consumption equal to 0.49.

[9]For example, nominal asset pricing models that have found success in matching the empirical properties of U.S Treasuries include Wachter (2006), Piazzesi and Schneider (2007), Gallmeyer, Hollifield, Palomino, and Zin (2007), and Bansal and Shaliastovich (2013).

[10]Other important studies of investor heterogeneity have focused on labour income shocks (Constantinides and Duffie (1996)), preferences (Wang (1996), Chan and Kogan (2002), and Bhamra and Uppal (2014)), and beliefs with and without frictions (Gallmeyer and Hollifield (2008), Buraschi, Trojani, and Vedolin (2013, 2014), Andrei and Hasler (2014), and Piatti (2015)).

provides information on real growth, Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2013) study the impact of disagreement about inflation when agents have 'Catching up with the Joneses' habit preferences, and find empirically that inflation disagreement raises long term bond yields. In contrast, Hong, Sraer, and Yu (2014) present contradictory evidence that inflation disagreement lowers long term yields, consistent with a model where Treasury market investors are subject to short sale constraints. Different than the work of these authors, we find that in a frictionless economy, even when agents have time-separable preferences, low risk aversion can generate risk reallocation that has significant asset pricing implications. Moreover, we present empirical evidence that shows the economic importance of belief dispersion about real growth.[11]

Finally, this paper relates to the literature on delegated portfolio management and the role of financial intermediaries in capital markets. When financial institutions play an important role in capital markets, their incentives structure may ultimately affects asset prices. A significant literature argues that when delegated managers earn convex performance-based incentives or do not fully bear the consequences of their decisions, moral hazard and risk shifting may emerge in equilibrium (Carpenter (2000), Panageas and Westerfield (2009) , Buraschi, Kosowski, and Sritrakul (2014)). As a consequence, asset prices should be studied in the context of a marginal agent with a low level of risk aversion. Unfortunately, the lion share of asset pricing models require large as opposed to low risk aversion. This paper shows that speculation among heterogeneous low risk aversion agents can play a key role in resolving this tension in the asset pricing literature.

I. Heterogeneous Belief Economies

Heterogeneous beliefs models depart from the traditional models by assuming that agents disagree on some features of the conditional distribution of the fundamentals. Rational or irrational motives can lead investors to such disagreement. However, regardless of the reasons for disagreement, the key insight of this literature is that, if agents can trade, the equilibrium pricing kernel is affected by heterogeneity in beliefs. In this section we discuss testable implications of this literature in the context of bond market.

[11] An additional important reduced-form contribution studying the belief impact on bond markets is made by Giacoletti, Laursen, and Singleton (2015) who propose a dynamic arbitrage-free model with Bayesian learning that incorporates priced dispersion in beliefs. The authors show that incorporating belief dispersion results in substantial outperformance of consensus forecasts over the past twenty-five years.

A. Differences in Belief

Consider two agents, a and b, each representing its own class with separate (absolutely continuous) subjective probability measures on the data generating process, denoted as $d\mathcal{P}_t^a$ and $d\mathcal{P}_t^b$. The difference in beliefs between the two agents can be conveniently summarized by the Radon-Nikodym derivative $\eta_t = \frac{d\mathcal{P}_t^b}{d\mathcal{P}_t^a}$, so that for any random variable X_t that is $\mathcal{F}_t$-measurable,

$$E^b(X_T|\mathcal{F}_t) = E^a\left(\eta_T X_T|\mathcal{F}_t\right), \qquad \text{with } \eta_t = 1. \tag{1}$$

In the literature, the Radon-Nikodym derivative η_t is either assumed as an exogenous process or obtained as the outcome of an optimal learning problem in which agents have different prior beliefs.[12] Independent of its micro-foundations, disagreement among agents affects the distribution of consumption in equilibrium, for $T < \infty$. Agents trade to equate ex-ante expected marginal utility of consumption, $E_t^a u'(C_T^a) = E_t^b u'(C_T^b)$. Thus, using (1), any frictionless equilibrium requires that $E_t^a u'(C_T^a) = E_t^a(\eta_T u'(C_T^b)$ so that innovations in η_t necessarily imply a different allocation of state-contingent consumption C_t^a and C_t^b between the two agents. Optimists will trade to shift consumption to states of the world in which their subjective probabilities are the highest, in exchange for a lower consumption in those states they deem less likely.

Indeed, in equilibrium, agents must have different individual stochastic discount factors, which need to satisfy $\Lambda_t^a = \eta_t \Lambda_t^b$. Thus, marginal utilities are not equated in each state and the dynamics of η_t directly affects heterogeneity in individual consumption growth. Recent evidence find that per capita volatility of consumption growth is much higher than aggregate consumption growth (Attanasio, Banks, and Tanner (2002) report that the first is up to twelve times larger; see Constantinides and Duffie (1996) for asset pricing implications). An important stream of the literature tries to reconcile this gap introducing incomplete markets, where the lack of perfect insurability induce idiosyncratic shocks (such as labor income shocks) to have an impact on individual marginal utilities. Heaton and Lucas (1996) find, however, that these models can reproduce a realistic equity premium only if transaction costs on trading financial assets are large enough to

[12] Example includes Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), who study economies in which the process η_t arise from investors' different prior knowledge about the informativeness of signals and the dynamics of unobservable economic variables. See Kurz (1994) for a discussion on the micro-foundations of disagreement. Dumas, Kurshev, and Uppal (2009) label the process η_t 'sentiment'.

significantly limit trade. They conclude that the size of these frictions are unrealistically large.

In economies with differences in beliefs, the wedge between agents marginal utilities is generated by shocks to their beliefs. Thus, limits to trading is not a required condition to obtain asset pricing implications. On the contrary, shocks to η_t will induce agents to trade more. Moreover, even after trading a complete set of contingent claims, agents still face the risk of future changes in other agents beliefs, so that trading does not reduce risk premia. Quite the opposite: it increases it.

B. Fundamentals

We study an endowment economy where a single consumption good and the nominal price level evolve according to

$$dC_t/C_t = g_t dt + \sigma_c dW_t^c, \tag{2}$$

$$dQ_t/Q_t = q dt + \sigma_q dW_t^q. \tag{3}$$

where we include a 'long run risk' component in consumption growth

$$dg_t = \kappa_g \big(\theta - g_t\big) dt + \sigma_g dW_t^g, \tag{4}$$

constant inflation rate, and correlation between shocks given by $\langle dW_t^c, dW_t^g \rangle = \rho_{c,g} dt$, $\langle dW_t^q, dW_t^g \rangle = \rho_{q,g}$ and $\langle dW_t^c, dW_t^q \rangle = \rho_{c,q}$. Now, consider two agents, each representing their own class, that have common information sets and 'agree to disagree' about how to process information. Mathematically, agents have different filtered probability spaces $\{\Omega, \mathcal{F}_t, \mathcal{P}(\Theta^i)\}$, where agent specific economic parameters that determine subjective measures $\mathcal{P}^i$ are contained in Θ^i. Since C_t is common and observable, consistent perceptions require that

$$dW_t^{c,i} = \sigma_c^{-1} \left(dC_t/C_t - g_t^i dt \right) \tag{5}$$

$$= dW_t^c + \sigma_c^{-1} \left(g_t - g_t^i \right) dt = dW_t^c + \mathcal{E}_t^i dt \tag{6}$$

where we have defined standardised forecast error of agent i as $\mathcal{E}_t^i = \sigma_c^{-1}\left(g_t - g_t^i\right)$. Since the above holds for both agents, subjective innovations are related by

$$dW_t^{c,b} = dW_t^{c,a} + \sigma_c^{-1}\left(g_t^a - g_t^b\right)dt = dW_t^{c,a} + \Psi_t dt \tag{7}$$

which defines the standardised 'disagreement' process Ψ_t. Model disagreement leads to different empirical likelihoods for the two agents. The difference in likelihood is summarized by the Radon-Nikodym derivative whose solution is given by[13]

$$\eta_t = \eta_0 \exp\left(-1/2\int_0^t \Psi_t^2 ds - \int_0^t \Psi_s dW_s^a\right) \tag{8}$$

Intuitively, the Radon-Nikodym derivative encodes the difference in beliefs between agents by assigning a higher (lower) weight for states of nature which agent b deems more likely (unlikely). Intuitively, if agents b believes consumption is likely to be smaller tomorrow than agent a does, η_T will be larger in down states.

C. Bond Prices

The dynamic properties of asset prices depend on the characteristics of the stochastic discount factor of the representative agent. In complete markets, Basak (2000) extends Cuoco and He (1994) approach to show how the competitive equilibrium solution can be obtained from the solution of a central planner problem.[14] Indeed, a representative investor utility function can be constructed from a (stochastic) weighted average of each individual utilities. With CRRA preferences individual consumption policies c_t^i and the stochastic discount factor Λ_t are given by

$$c_t^a = \frac{C_t}{1+\eta_t^{1/\gamma}} \quad , \quad c_t^b = C_t \frac{\eta_t^{1/\gamma}}{1+\eta_t^{1/\gamma}} \tag{9}$$

[13] which exists so long as Novikov's condition holds $E_t^a \exp\left(\frac{1}{2}\int_0^t \Psi_s^2 ds\right) < \infty$.

[14] Constantinides (1982) extends Negishi (1960)'s results and proves the existence of a representative agent with heterogeneous preferences and endowments but with homogeneous beliefs. In an incomplete market setting with homogeneous agents Cuoco and He (1994) show a representative agent can be constructed from a social welfare function with stochastic weights. Basak (2000) discuss the aggregation properties in economies with heterogeneous beliefs but complete markets. He shows that a representative can be constructed from a stochastic weighted average of individuals marginal utilities.

$$\Lambda_t = e^{-\delta t} C_t^{-\gamma} \left(1 + \eta_t^{1/\gamma}\right)^{\gamma} \tag{10}$$

Equation 9 shows that under heterogeneous beliefs disagreement has direct implication on equilibrium individual consumption. The resulting stochastic discount factor, Λ_t, is the product of the stochastic discount factor of a homogeneous CRRA economy Λ_t and a function η_t. Indeed, most models discussed in the literature give rise to equilibrium discount factors that can be cast in this form.

Then, the price of an T-period default free nominal zero-coupon bond is given by

$$B^{\$}(\tau) = E_t^i \left[e^{-\delta\tau} \left(\frac{C_T}{C_t}\right)^{-\gamma} \left(\frac{Q_T}{Q_t}\right)^{-1} \left(\frac{1 + \eta_T^{1/\gamma}}{1 + \eta_t^{1/\gamma}}\right)^{\gamma} \right] \tag{11}$$

Solving for bond prices require solving for the conditional expectation of the product of two terms. The first emerges in the traditional homogeneous case; the second one arises because of the impact of disagreement on the ex-post redistribution of wealth.

Define $X_t = \log C_t$, $Y_t = \log Q_t$, and $Z_t = \log \eta_t$. For integer risk aversion, one could binomial expand $(1 + e^{\frac{1}{\gamma} Z_T})^{\gamma}$ resulting in a sum of exponential functions that can be solved in closed form. However, since we want to study the implications also when a significant set of agents have $\gamma < 1$, such as intermediaries and hedge funds, this approach cannot be applied.[15] For arbitrary risk aversion, one cannot take the transform of the SDF directly because it is not a tempered distribution.[16] However, expanding around the integer case leaves a residual component that can be transformed after applying an appropriate damping factor. The final result is a sum of characteristic functions of the form

$$\phi(\tau; u) = E_t^i \left(e^{u_1 X_T + u_2 Y_T + u_3 Z_T}\right). \tag{12}$$

From Feynman-Kac we know that the solution of the previous probabilistic problem satisfies a partial differential equation with an appropriate boundary condition. Following Cheng and Scaillet (2007) we conjecture the solution is affine in the extended state vector $V_t = (X_t, Y_t, Z_t, \widehat{g}_t^a, \Psi_t, \Psi_t^2)$.

[15] The solution given by Xiong and Yan (2010) only applies to the case of log utility investors. Dumas, Kurshev, and Uppal (2009) derive solutions for the optimal holdings of risky assets for γ integers using binomial expansions of the Fourier transform. These methods do not allow for an analytical solution for the $\gamma < 1$ case, which is the focus of this paper.

[16] See Chen and Joslin (2012) for a discussion of the class of functions that admit generalised transforms.

Solving the resulting partial differential equation, we can recover the characteristic function in terms of a set of separable ordinary differential equations.

Theorem 1 (Bond Prices)**.** *The term structure of bond prices is a weighted sum of exponentially affine functions that depend on growth rate dynamics, differences in beliefs, and the distribution of wealth.*

$$B^{\$}(\tau) = e^{-\delta\tau}(1+\eta_t^{1/\gamma})^{-\gamma} \sum_{j=0}^{\lfloor\gamma\rfloor+1} \binom{\lfloor\gamma\rfloor+1}{j} \frac{\gamma}{\pi} \int_0^{+\infty} Re\left[\eta_t^{u_3}\phi(\tau;u)\frac{\Gamma[g_1]\Gamma[g_2]}{\Gamma[g_1+g_2]}\right] dk \tag{13}$$

where $\Gamma[\cdot]$ *is the (complex) gamma function,* $\lfloor\cdot\rfloor$ *is the floor operator, and*

$$\begin{aligned} \alpha &= \lfloor\gamma\rfloor + 1 - \gamma \quad , \quad g_1 = \alpha/2 - i\gamma k \quad , \quad g_2 = \alpha/2 + i\gamma k \\ u_1 &= -\gamma \quad , \quad u_3 = (2j-\alpha)/2\gamma - ik, \end{aligned}$$

$\phi(\tau;u) = e^{\alpha(\tau;u)+\beta(\tau;u)'v}$ *and* $\{\alpha(\tau),\beta(\tau)\}$ *are functions of time and the structural parameters of the economy.*

Proof: See Appendix. Given this solution, remaining term structure quantities of interest are immediate. For instance, bond sensitivities to fundamentals shocks are given by

$$\left[\sigma_{b,c}^{\tau}\ ,\ \sigma_{b,q}^{\tau}\right] = \frac{1}{B^{\$}(\tau)}\left[\frac{\partial B^{\$}(\tau)}{\partial \widehat{g}^a}\ ,\ \frac{\partial B^{\$}(\tau)}{\partial \eta}\ ,\ \frac{\partial B^{\$}(\tau)}{\partial \psi}\right] \begin{bmatrix} \sigma_{c,g} & \sigma_{q,g} \\ -\Psi_t & 0 \\ \sigma_{c,\Psi} & \sigma_{q,\Psi} \end{bmatrix} \tag{14}$$

which can be computed in semi-closed and given in the appendix. Notice, in heterogeneous agent models volatility depends explicitly on η_t and Ψ_t and since the term structure is quadratic in Ψ_t^g bond sensitivities to $d\widehat{W}_t^{c,i}$ shocks are stochastic. See appendix for computational details.

II. Short Term Yields

Applying Itô's lemma to equation 10 the equilibrium short rate is given by

$$r_t = \underbrace{\delta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2}_{\text{Lucas-Tree Terms}} + \gamma\underbrace{(\omega^a(\eta_t)\widehat{g}_t^a + \omega^b(\eta_t)\widehat{g}_t^b)}_{\text{Consensus Bias}} + \underbrace{\frac{\gamma-1}{2\gamma}\omega^a(\eta_t)\omega^b(\eta_t)\Psi_t^2}_{\text{Speculative Demand}} \tag{15}$$

where $\omega^i(\eta_t) = c_t^i/C_t$ is investor's i consumption share.

When disagreement is zero the short term interest rate is given by the Lucas solution ($r_t = \delta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + \gamma\widehat{g}_t$). In the heterogeneous case, the short term interest rates includes two new terms. The first is ($\left[\omega_t^a(\eta_t)\widehat{g}^a + \omega^b(\eta_t)\widehat{g}_t^b\right]$) is a wealth effect. The larger the expected growth opportunity, the higher the demand for current consumption, the lower the demand for savings, thus the higher the interest rate. However, when $\eta_t \neq 1$, this term differs from the consensus belief $\frac{1}{2}\widehat{g}_t^a + \frac{1}{2}\widehat{g}_t^b$. Speculative activity undertaken in the past affects agents' relative wealth today and this term biases the short rate towards the belief of the agent who has been relatively more successful.[17] The implications of this term for the term structure are rich. For example, the short rate, and hence the entire yield curve, becomes path dependent even though state dynamics are Markovian. Moreover, if the path of the economy were such that the distribution of wealth was shifted towards pessimists (optimists) the bond prices will be inflated (deflated) with respect to their homogeneous counterparts. The second term is due to speculative demand and is given by the product of $\omega^a(\eta_t)\omega^b(\eta_t)$ and Ψ_t^2.

In homogeneous Lucas economies the model-implied risk-free rate is substantially higher than what is observed empirically (a tension that is commonly referred to as the '*risk-free rate puzzle*'). To visualise the interaction of risk aversion, expectations, and relative wealth, consider the

[17] Jouini and Napp (2006) also construct a consensus investor whose stochastic discount factor contains an aggregation bias.

sensitivity of the short rate with respect to the state vector $[\widehat{g}_t^a,\ \eta_t,\ \Psi_t]$:[18]

$$\frac{\partial r}{\partial \widehat{g}^a} = \gamma = \frac{1}{EIS} \tag{16}$$

$$\frac{\partial r}{\partial \Psi} = -\gamma\sigma_c\left(\frac{\eta^{1/\gamma}}{1+\eta^{1/\gamma}}\right) + \left(\frac{\gamma-1}{\gamma}\right)\frac{\eta^{1/\gamma}}{(1+\eta^{1/\gamma})^2}\Psi_t \tag{17}$$

$$\frac{\partial r}{\partial \eta} = \sigma_c\left(\frac{\eta_t^{1/\gamma}}{(1+\eta_t^{1/\gamma})^2}\right)\Psi_t - \frac{1}{2}\frac{\eta_t^{\frac{1-\gamma}{\gamma}}}{(1+\eta_t^{1/\gamma})^3}\left(\eta_t^{\frac{1}{\gamma}}-1\right)\left(\frac{1}{\gamma}\right)\left(\frac{\gamma-1}{\gamma}\right)\Psi_t^2 \tag{18}$$

Figure 2 summarizes the results. The left panel shows that when $\gamma < 1$ the substitution effect dominates and short term interest rates are negatively related to disagreement, $\partial r/\partial\Psi < 0$. This is due to the second term in equation (17).

The risk-free rate also depend explicitly on η_t. The right panel shows the sensitivity of the short rate with respect to η for different levels of ω_t^a. Speculative demand introduces a non-linearity which is asymmetric with respect to the distribution of wealth. When the economy is dominated by pessimists ($\omega_t^a = 0.25$), shocks that further increase the wealth of these agents (negative $d\widehat{W}_t^{c,a}$ realisations) increase the short rate through its dependence on $\partial r/\partial\eta$. When the economy is dominated by optimists, on the other hand, shocks that increase the wealth of these agents (positive $d\widehat{W}_t^{c,a}$ realisations), can increase or decrease the short rate depending on the knife edge restriction that risk aversion is above or below one (see also figure 4).

[Insert figure 2 about here]

The implication of a decreasing risk free rate and increasing bond risk premia for increasing disagreement when $\gamma < 1$ implies that positive shocks to disagreement can increase the slope of the yield curve.

To understand the intuition for the effect of this term consider the diffusion for the relative

[18]Short rate sensitivities are obtained by rearranging the risk free rate as

$$r_t = \delta - \frac{1}{2}\gamma(\gamma+1)\sigma_C^2 + \gamma g^a - \psi_t\left(\frac{\eta^{1/\gamma}}{1+\eta^{1/\gamma}}\right)\left(\gamma\sigma_D - \frac{\gamma-1}{2\gamma}\frac{1}{1+\eta^{1/\gamma}}\psi_t\right)$$

wealth of agent a[19]

$$d\omega_t^a = \underbrace{\frac{\gamma-1}{2\gamma}\omega_t^a\omega_t^b\Psi_t^2}_{\text{Speculative Demand}}\left(\frac{(\gamma-1)+2\gamma\omega_t^b}{\gamma(\gamma-1)}\right)dt + \frac{1}{\gamma}\omega_t^a\omega^b\Psi_t d\widehat{W}_t^{c,a} \tag{19}$$

from which we identify the speculative demand in the drift of individual consumption.[20] The reason is because an increase in Ψ_t changes the investment opportunity set, as it increases speculative opportunities between agents. The sign of the effect depends on whether γ is greater or smaller than 1. For $\gamma > 1$ the wealth effect dominates: speculation raises the drift of planned consumption, which is fixed today; thus, interest rates must rise to clear the market. When $\gamma < 1$ the substitution effect dominates: speculation increases expected returns raising the price of current consumption relative to future consumption, lowering the drift of planned consumption; thus, interest rates must fall.

III. Long Term Yields

A. Learning

The central contribution of this paper is to quantify the impact of heterogeneous beliefs on the term structure of interest rates. While the previous analysis is general to the class of power utility preferences, to allow for an assessment of long term yields one needs to make assumptions about agents' learning process. It is common in this literature to assume that agents learn from identical information sets that include realisations of consumption and the price level, which is correlated with stochastic growth: $\mathcal{F}_t = \{C_\tau, Q_\tau\}_{\tau=0}^t$. Denote agent i's conditional forecast $\widehat{g}_t^i = E_t^i\left[g_t|\mathcal{F}_t, \Theta^i\right]$ and posterior variance $\nu_t^i = E_t^i\left[(\widehat{g}_t^i - g_t)^2|\mathcal{F}_t, \Theta^i\right]$, where Θ^i is a set of subjective model parameters.

The set of subjective models we consider differ along two dimensions. First, agents may have different models for the correlation between consumption and growth rate shocks ($\rho_{c,g}^i$), and consumption and price level shocks ($\rho_{q,g}^i$). These parameters play an important role for term structure properties in homogeneous agent economies since they determine the extent to which bonds are risky bets or hedging instruments against consumption and inflation shocks. Second,

[19] Add note to explain where the relative wealth diffusion comes from.

[20] An analogous diffusion is obtained for agent b under his measure.

agents may hold specific beliefs about the long-run consumption growth rate ($\theta^a \neq \theta^b$). Indeed, a significant stream of the empirical asset pricing literature argue about the existence of significant challenges in measuring the long-run properties of the economy.[21] Since yield curve implications of these parameters are rather different, we study and compare their different roles when agents disagree about them.

Since state dynamics are conditionally Gaussian, standard linear filtering results applied to the model (23)-(5) give rise to a closed-form solution for the optimal posteriori mean and variance of each agent i, whose dynamics satisfy the following condition (see Appendix for details)

$$d\widehat{g}_t^i = \kappa_g(\theta^i - \widehat{g}_t^i)dt + \left(\frac{\nu^{*,i} + \rho_{c,g}^i \sigma_c \sigma_g}{\sigma_c}\right) d\widehat{W}_t^{c,i} + \rho_{q,g}^i \sigma_g d\widehat{W}_t^{q,i}, \tag{20}$$

$$\nu^{*,i} = -\kappa_g \sigma_c^2 - \rho_{c,g}^i \sigma_c \sigma_g \sigma_c \sqrt{\kappa_g^2 \sigma_c^2 + 2\kappa_g \rho_{c,g}^i \sigma_c \sigma_g + \sigma_g^2 (1 - (\rho_{q,g}^i)^2)}. \tag{21}$$

Proof: See Appendix.

Using these optimal learning conditions, standardised disagreement, $\Psi_t = \sigma_c^{-1}(\widehat{g}_t^a - \widehat{g}_t^b)$, satisfies:

$$\begin{aligned} d\Psi_t &= \frac{\kappa\sigma_c + \sigma_{c,g}^b}{\sigma_c}\left(\left(\frac{\kappa(\theta^a - \theta^b)}{\kappa\sigma_c + \sigma_{c,g}^b}\right) - \Psi_t\right) dt + \left(\frac{\sigma_{c,g}^a - \sigma_{c,g}^b}{\sigma_c}\right) dW_t^{c,a} + \left(\frac{\sigma_{q,g}^a - \sigma_{q,g}^b}{\sigma_c}\right) dW_t^q \\ &= \kappa_\Psi \left(\theta_\Psi - \Psi_t\right) dt + \sigma_{c,\Psi} dW_t^{c,a} + \sigma_{q,\Psi} dW_t^q. \end{aligned} \tag{22}$$

Disagreement about θ and ρ has different effects on the dynamics of Ψ_t. When $\theta^a \neq \theta^b$, the disagreement process has a *non-zero* long run mean both conditionally and unconditionally. In this case, the disagreement process does not revert to zero in steady state. Moreover, conditional disagreement can take both positive and negative values: growth rate optimists can become growth rate pessimists and vice-versa, depending on the realization of the state variables. Equation (22) shows that disagreement about ρ mainly has an impact on the volatility of disagreement and it affects the steady state mean of Ψ_t if and only if $\theta^a \neq \theta^b$. It is reasonable to conjecture that disagreement about θ plays a key role for the equilibrium properties of long term bonds while

[21] Hansen, Heaton, and Li (2008) argue that econometricians face severe measurement challenges when quantifying the long-run components of the economy. For a related discussion see Pastor and Stambaugh (2000) who study the statistical properties of predictive systems when the predictors are autocorrelated but κ_g is not known. Chen, Joslin, and Tran (2012) argue that difficult to measure parameters of the economy, such as the likelihood of rare disasters, are a natural source of disagreement.

disagreement about ρ affects the short-end of the curve and yield volatilities.

B. *Parameter Values*

To study the interaction of different learning and disagreement environments with different levels of risk aversion, we calibrate the model using empirical moments of realised aggregate consumption and utilise survey data on differences in belief to study the model-implied risk-free rate, level and shape of the yield curve, bond yield volatilities, and bond risk premia.

To calibrate the parameters, we adapt the approach of Schorfheide, Song, and Yaron (2014) who address concerns about calibrating consumption dynamics at annual frequencies in models where the decision intervals (in discrete time) are monthly (Beeler and Campbell (2009)). First we obtain monthly and annual estimates for the mean, volatility, first and second autocorrelations for real per-capita consumption growth for non-durables and services for the sample period January 1959 - January 2015.[22] We set $\bar{\theta} = 1.96\%$ to match the sample mean of real consumption growth at annual frequency. The persistence of consumption growth is set to $\kappa_g = -\ln(0.98) \times 12$, to match the annualised continuous time equivalent of Schorfheide, Song, and Yaron (2014). We set the correlation between consumption shocks and growth rate shocks equal to zero. Next, given an initial guess for σ_g we implement a grid search on σ_c to match the time-aggregated annualized volatility, AR(1) and AR(2) coefficients of annual real per-capita consumption growth. Then, given this estimate for σ_c, we solve for σ_g to match the monthly volatility, AR(1), and AR(2) coefficients. Finally, we iterate between targeting annual versus monthly consumption dynamics until convergence.[23] For the inflation dynamics, we match the monthly mean growth rate and volatility of the consumer price index for the overlapping sample period. Finally, we set the rate of time preference to $\delta = -\log(0.999)$.[24] The calibrated parameters are reported in table I and the model implied versus empirical estimates at both annual and monthly frequency are reported in table II. The results suggests that this approach allows indeed to closely match first and second moments of real consumption at both monthly and annual frequencies.

[Insert table I and table II here]

[22] We obtain data from the U.S. Bureau of Economic Analysis (BEA). To compute annual and monthly real per capital consumption growth on non-durables are services we use NIPA Tables 2.3.3, 2.8.3, and 7.1.

[23] This procedure is implemented on discretised real consumption growth given by equations and .

[24] This parameter simply affects the average level of the term structure.

The calibration of the model also requires assumptions about the average disagreement on log-run growth θ and instantaneous correlation coefficient $\rho_{c,g}$. We use BlueChip survey data and estimate that the average disagreement on long-term GDP growth at a 10 year horizon is equal to 1%.[25] Figure 7 shows that while long-run disagreement is smaller than its short-run counterpart, its magnitude is quite significant, ranging between 0.8% and 2.3%.[26] We use the observed average long-run disagreement to calibrate an economy where the two classes of agents are equally distant from the true θ^o, with $\theta^{a,b} = \theta^o \pm 0.5\%$. Similarly, we assume equal distance in agents beliefs about the instantaneous drift g_t, $\Psi \equiv \sigma_c^{-1}\left(\widehat{g}_t^a - \widehat{g}_t^b\right)$.

In a homogeneous agent Lucas economy with CRRA preferences, the correlation coefficient $\rho_{c,g}$ determines whether the real risk free rate is pro- or counter-cyclical. Thus, $\rho_{c,g}$ determines whether bonds are risky bets or hedges. There is limited consensus in the literature about the average value $\rho_{c,g}$. We follow Bansal and Yaron (2004) and set the objective correlation parameters as $\rho^o_{c,g} = \rho^o_{q,g} = 0$. Then, we assume that agents disagreement about this parameter is $\rho^a_{c,g} = +0.5$ and $\rho^b_{c,g} = -0.5$, which implies disagreement about bond risk premia. Agent a (b) believes bond risk premium is negative (positive). Disagreement about the sign of the bond risk premium becomes a natural channel supporting positive trade in equilibrium. The remaining and complete set of parameters used in the calibration is reported in table I.

C. Main Result

Table III summarises results for the level (average yield between 1 to 5 years), slope (5 year minus 1 year yield), the three month yield, the volatility of yields for maturities of 1 and 5 years, and excess returns on bonds with maturity 2 and 5 years. The table summarises results both for an economy with $\gamma = 0.75$ (Panel A, C, and E) and $\gamma = 2$ (Panel B, D, and F). Given that market realizations shift wealth across agents with endogenous implications on the equilibrium term structure, for each level of risk aversion we reports implications depending on agents relative wealth: $\omega_a = 0.50$ (Panel A, symmetric economy), $\omega_a = 0.75$ (Panel C, optimistic wealth bias), and $\omega_a = 0.25$ (Panel E, pessimistic wealth bias).

Summarising the central message, it is well known that traditional homogeneous economies

[25]This is obtained from the difference between top and bottom decile of the cross-sectional distribution of forecasts.

[26]See also Andrade, Crump, Eusepi, and Moench (2014).

with zero disagreement find it difficult to match key moments in bond markets, which has motivated an important literature in asset pricing. Table III, panel B, shows that for high levels of risk aversion larger disagreement make these puzzles even worse. Indeed, when $\gamma = 2$ belief heterogeneity decreases the model ability to match, at the same time, the risk free rate, the slope of the yield curve, and the level of bond risk premia. These results apply independently of the distribution of relative wealth between optimists and pessimists, or the type of disagreement (panel D and F). However, when risk aversion is low ($\gamma = 0.75$) the interaction of an increase in disagreement and a skewed distribution of relative wealth ($\omega_a = 75\%$ and $\omega_a = 25\%$) increases both bond risk premia and the slope of the term structure without generating an excessively high risk free rate. Indeed, when $\omega_a = 75\%$ the impact of a disagreement equal to $\hat{g}^a - \hat{g}^b = 1\%$ on bond risk premia is quite significant. This is interesting since in homogeneous economies it is well known that matching bond risk premia comes at the cost of creating both excessive bond volatility and an untenable risk-free rate puzzle.

D. Discussion

In what follows we provide further economic intuitions about these results. Bond volatility is monotonically increasing in disagreement, becoming an important factor driving the quantity of risk. The economic mechanism is rather simple but different from traditional channels. When agents agree to disagree, they engage in beliefs-based trading. The larger the amount of trade, the larger the ex-post volatility of individual consumption, which implies a large volatility for discount bonds. The diffusion component of equation 19 makes this explicit:

$$d\omega_t^a - E_t^a[d\omega_t^a] = \frac{1}{\gamma}\omega_t^a\omega^b\Psi_t d\widehat{W}_t^{c,a} \tag{23}$$

Individual agent consumption volatility is higher for lower levels of risk aversion. Since agents are forward looking, equilibrium bond prices must discount larger individual consumption risks, thus requiring larger risk premia. Table III reports expected bond risk premia from the perspective of an unbiased econometrician endowed with the knowledge of the true data generating process (i.e.

he knows the true g_t) [see Appendix]

$$\mu^{\tau}_{\mathbb{P}^o} - \mu_{\mathbb{Q}} = \underbrace{\left[\gamma\sigma_c + \frac{1}{\sigma_c}\left[g_t - (\omega^a_t g^a_t + \omega^b_t g^b_t)\right]\right]}_{\text{PoR}} \times \underbrace{\sigma^{\tau}_{b,c}(\Psi_t, \eta_t))}_{\text{QoR}} \tag{24}$$

where $\mu^{\tau}_{\mathbb{P}^o}$ is the objective drift. Equation (24) shows that bond risk premia are equal to the product of two terms. The first term is given by the price of risk (PoR), which is equal to the sum of the traditional risk premium emerging in an homogeneous Lucas economy, namely $\gamma\sigma_c$, plus a term equal to the wealth-weighted belief bias, which can be rewritten as $\sum_{i=a,b} \omega^i_t \varepsilon^i_t$ with ε^i being agent i's standardised forecast error defined above. The second term is the sensitivity of bond prices to consumption shocks $\sigma^{\tau}_{b,c}(t)$, i.e. the quantities of risk (QoR). The product of PoR and QoR depend on the distribution of wealth $\omega(\eta)$, which in turn is a function of the full history of beliefs between time 0 and t as captured by η_t, and the current level of disagreement Ψ_t.[27] Thus, states of the world when bond risk premia significantly deviate from $\gamma\sigma_c$ are when (i) wealth-weighted beliefs are biased and (ii) the level of current disagreement induce sufficient trading to significantly increase the expectation of ex-post individual consumption shocks, thus increasing $\sigma^{\tau}_{b,c}(\Psi_t, \eta_t))$. When these conditions are met, bond risk premia can be large even if $\gamma\sigma_c$ is small. This is juxtapose to the CRRA representative agent paradigm which requires unrealistically large levels of risk aversion to reproduce bond risk premia comparable to the data. At the same time, we find that low levels of risk aversion can match expected bond risk premia without incurring the cost of unrealistic second moments of bond yields.

To understand further the role played by the skewness of the wealth distribution, it is useful to split the sensitivity of bond returns to dW^c_t shocks (i.e. QoR) into the two components due to η_t and Ψ_t. Figure 4 summarizes the result. For large values of ω_a, both PoQ and PoR are negative. The dominant component of the QoR is played by the dependence on η_t and the product of PoQ and PoR is positive and large in magnitude. For low values of ω_a, on the other hand, the PoR is positive and the dominant component of the QoR is played by the dependence on Ψ_t.[28] In both cases, the product of PoR and QoR generates positive bond risk premia, however bond risk premia

[27] This implies that in the risk premium is state dependent. To see this, note that two economies with the same vector $[g^a_t, \psi^g_t]$ at time t have different risk premia depending on which agent has accumulated more wealth

[28] To see this, compare the magnitudes of panel (a) versus panel (b).

are smaller in magnitude for small values of ω_a due to the role played by η_t on the QoR. When $\omega_a = 0.25$ and $\gamma = 0.75$, the level of the bond risk premia drop in magnitude but remain positive for disagreement levels $\hat{g}^a - \hat{g}^b > 0.50\%$.

To summarize, positive bond risk premia arise in heterogeneous agent economies with risk tolerant (low risk aversion) agents when market realisations generate a skew in the distribution of wealth.

[Insert figure 4 about here]

Finally, Table III part II compare the impact of different learning environments that may result in either disagreement about both long-run growth rates θ or about shocks correlation $\rho_{c,g}$ (i.e. whether bonds are hedges or risky assets). The question is clearly a very important one, given both the significant literature on long-run risk in asset pricing and the literature on heterogenous beliefs, which usually assumes disagreement about $\rho_{c,g}$. Panel A and B compare risk premia in an economy with a wealth-weighted bias toward larger long-run growth rates (i.e. $\omega^a = 0.75$ and $\gamma = 0.75$) when $\rho^a > \rho^b$ (panel A) and $\rho^a < \rho^b$ (panel B). Consistent with economic intuition, when a larger economic proportion of agents believe bonds are hedges (panel A), risk premia are lower. Panel C and D report the results in the case of a pessimistic economy with $\omega^a = 0.25$. When a larger wealth-weighted proportion of agents have pessimistic beliefs about growth rate and consider bonds hedges (panel D), bond risk premia can become negative. This accords to economic intuition. Overall, however, the main take away is that when we compare the two different channels of disagreement, the impact of disagreement on long-run growth is significantly larger than on correlation. Indeed, risk premia in panel A are similar to those in panel B; similarly for panel C and D. On the other hand, risk premia are significantly diffferent when we compare economies with large disagreement about long-run growth (as shown by panels B and C). This is particularly interesting given the intrinsic uncertainty about the estimation of long-run quantities and suggests the importance of modelling disagreement on long-run growth rates.

IV. The Role of γ vs EIS

Mehra and Prescott (1985) show that in order to reconcile large average stock returns with smooth consumption growth the representative agent must be endowed with large risk aversion. This is commonly known as the *equity premium puzzle*. An extensive literature, however, shows that large risk aversion generates a real risk free rate that is too large to be empirically plausible.[29] This is commonly known as the *risk free rate puzzle* (Weil (1989)). Large risk aversion coefficients are problematic for two additional reasons. The first relates to results in behavioural economics which use risk elicitation methods in laboratory experiments and conclude that while there is systematic evidence that subjects in laboratory experiments behave as if they are risk averse, the degree of risk aversion is modest (see Harrison and Rutström (2008) for a recent survey article).

The second reason relates to the large literature studying moral hazard in financial institutions. Indeed, when delegated managers earn convex performance-based incentives or do not fully bear the consequences of their decisions, they may have the incentive to engage in risk shifting.[30] In aggregate, their behaviour may appear as motivated by low risk aversion. Indeed, the results in Table I are consistent with the notion of a marginal agent with low but positive risk aversion (i.e. $\gamma = 0.75$), as suggested by the financial intermediation literature. Once this assumption is embedded in a heterogeneous beliefs model, this simple specification can resolve a number of bond pricing puzzles.

When preferences are CRRA, $EIS = 1/\gamma$, thus the calibration suggests that an EIS equal to $1/0.75 = 1.33$ is consistent with the term structure moments considered here. There is little empirical consensus about the correct value of EIS and it is widely acknowledged that measuring EIS is plagued with econometric difficulties, being sensitive to assumptions about expectation formation, consumption measurement and asset market participation.[31] Indeed, Havranek, Horvath, Irsova, and Rusnak (2015) conduct a large scale meta-analysis survey of the literature and report a mean estimate of 0.5 with a standard deviation of 1.4. Our value is within their one standard deviation bounds. Bansal and Yaron (2004) assume the EIS to be equal to 1.5, arguing that previous instrumental variables are sensitive to the assumption of whether con-

[29]For a detailed discussion of the performance of consumption based asset pricing see Campbell (2003)

[30]Carpenter (2000), Panageas and Westerfield (2009) compute the optimal portfolio choice of a manager with performance fees that resemble a call options. This makes them less risk averse and, in some cases, risk seeking. When these institutions are also subject to withdrawals and termination of short-term credit lines, Buraschi, Kosowski, and Sritrakul (2014) argue that effective risk aversion is still positive but small in magnitude.

[31]Using disaggregated consumption data Attanasio and Weber (1993) and Beaudry and Van Wincoop (1996) find values of ψ closer to 1. Using the U.S. Consumer Expenditure Survey Vissing-Jorgensen (2002) estimates a ψ between $0.3 - 0.4$ for stockholders and around $0.8 - 1.0$ for bondholders. Using the Survey of Consumer Expectations Crump, Eusepi, Tambalotti, and Topa (2015) obtain estimated close to 0.80 for the general population which rise to above 1.00 for high income individuals.

sumption growth and asset returns are homoskedastic.[32] Thus, while the focus of this paper is on the link between belief heterogeneity, trade, and risk tolerance (low risk aversion) we do not believe our implied values of EIS are unreasonable given errors in measurements or commonly employed values in the asset pricing literature.

Lower values of EIS have been partially a response to Weil (1989) who observed *'why is it, if consumers are as averse to intertemporal substitution as some estimates suggest, that the risk-free rate is so low?'*. Indeed, in the context of single agent CRRA economy, Hall (1988) assumes a very low level of EIS partially because of the observed low level of risk-free rates. Table I suggests that heterogeneity in disagreement helps matching the level of the risk free rate even without assuming either extremely low levels of EIS or introducing Epstein-Zin preferences to relax the link between between EIS and γ.

V. Survival

A long history in economics invokes market selection arguments in defence of rational expectations. In the context of profit maximising firms, Friedman (1953a) writes "given natural selection, acceptance of the [rational expectations] hypothesis can be based largely on the judgment that it summarizes appropriately the conditions for survival" while Alchian (1950) writes "As in a race, the award goes to the relatively fastest". The main argument of these early proponents is that the economy should eventually make rational maximising decisions since those who behave irrationally will be driven out of the market by those who behave *as if* they were rational.

More recently, the market selection hypothesis has been called into question. In a simple overlapping generations economy, De Long, Shleifer, Summers, and Waldmann (1990) argue that irrationally optimistic noise traders can dominate asset markets. Kogan, Ross, Wang, and Westerfield (2006) study an economy where investors solve a portfolio choice problem without intermediate consumption, and show that a moderately optimistic investor can drive a rational investor out of the market if their relative risk aversion coefficient is larger than one. Sandroni (2000) and Blume and Easley (2002) show that this results does not carry over to general equilibrium when agents make both consumption and portfolio decisions. Extending these analyses Yan (2008)

[32] Time-varying second moments introduce a time-varying intercepts to instrumental variable regressions that are known to bias point estimates.

shows that natural selection may still fail in general equilibrium if irrational agents have larger savings motives than rational agents.

The previous sections argued that trading amongst investors with heterogeneous beliefs can have significant impact on asset prices. Moreover, this impact is increasing in risk tolerance because agents become more willing to speculate. This subsection studies the survival properties of this economy.

Computing the long run distribution of wealth requires inverting the characteristic function of η under the objective measure, which can then transformed to the distribution of ω_i. The appendix details the computations and figure 5 plots survival results for the case that (i) both agents hold equally incorrect beliefs; and (ii) one agent holds correct beliefs while the other agent is overly pessimistic.[33]

[Insert figure 5 about here]

The left panels of 5 plot the wealth distribution of agent a when both agents hold equally incorrect beliefs, as in the calibration above ($\theta^a = 1.96\% + 0.50\%, \theta^b = 1.96\% - 0.50\%$). The right panels of figure plot wealth distributions of agent a when she agent holds the correct belief ($\theta^a = \theta = 1.96\%$) and agent b believes incorrectly the long run growth rate of consumption is 1% below the objective measure ($\theta^b = 1.96\% - 1.00\%$). Top panels show wealth distributions after 25 years and bottom panels plot the wealth distributions after 50 years, sample periods that would be typical employed in empirical work. Each panel considers level of risk aversion equal to $\{0.80, 1.0, 2.0, 5.0\}$.

When both agents are equally wrong, one agent is a dogmatic optimist while the other agent is dogmatic pessimist. As we shall see below, dogmatic beliefs is consistent with the survey evidence from professional forecasters. In this case the distribution of wealth is, unsurprisingly, non-degenerate. More interestingly, consistent with the arguments made above, for large levels of risk aversion this economy looks more than a homogeneous economy than its low risk aversion counterpart: the distribution of wealth has a low variance. The reason for this is that the volatility of individual consumption is inversely proportional to risk aversion because as agents become increasingly risk tolerance they are more willing to speculate on their beliefs. This implies a larger

[33] We omit results on excess optimism since they generate similar implications to excess pessism.

reallocation of wealth ex-post. However, for $\gamma = 0.80$ the distribution remain close to uniform so that both agents have equal probability of survival. Repeating the main result from table III, for $\gamma < 1$, and for states of the world where the distribution of wealth deviates from a $\omega^a = \omega^b$, we obtain a low risk free rate, positive slope of the term structure, and positive bond risk premia. For the case that agents make symmetric forecast errors, the survival result discussed here implies that such states have equal probability of being realised.

How robust are the long run asset pricing implications to the assumption of symmetric forecast errors? When agents have identical CRRA preferences, irrational agents vanish in the long run (Borovička (2015)). However, in practice natural selection may take a very long time since the process of wealth accumulation occurs very gradually (Yan (2008) and Dumas, Kurshev, and Uppal (2009)). While previous authors have highlighted this point for the case that hedging motivates dominate trade ($\gamma > 1$) nobody has analysed the speculative case ($\gamma < 1$). The right panels of 5 show that the screw in the distribution of wealth is indeed decreasing in risk aversion. For high levels of risk aversion there are only very gradual shifts in the wealth distribution. Speculative motives increase the likelihood that natural selection prevails. However, after 25-years the irrational agent still holds a significant proportion of the wealth distributions, and after 50-years the irrational agents still has a non-trivial share of wealth. This result highlights that even when agents are speculative in nature it make take a very long time for irrational investors to become extinct.

VI. Testable Implications

The results of the model calibration suggest a number of testable implications due to the interaction between disagreement, risk aversion, and the wealth-weighted distribution of beliefs. We focus on three predictions from our preferred calibration, which we test empirically using survey data from professional market forecasters' beliefs about the macroeconomy:

H_{01}**: The Short Term Interest Rate.** In equilibrium, the level of disagreement emerges as a state variable that affects the dynamics of short term interest rate. The sign of the effect depends on the relative importance of the wealth and substitution effects. When $\gamma > 1$, the model predicts that higher disagreement increases interest rates due to the dominating wealth effect. The opposite occurs when $\gamma < 1$.

H_{02}**: The Slope of the Yield Curve.** Disagreement Ψ_t also affects the shape of the term structure. First, in a symmetric economy when $\gamma < 1$ the slope of the term structure is increasing in disagreement; when $\gamma > 1$, the opposite is true and for large levels of disagreement the increase in short-term interest rates can induce negatively sloped yield curve. The effect of disagreement on the slope of the term structure results from two channels: (a) an impact on short term rates (H_{01}); and (b) a risk premium demanded for long term bonds (H_{03}). Thus, a joint implication of our preferred calibration is that, after controlling for consensus fundamentals, positive shocks to disagreement should raise the slope of the yield curve.

H_{03}**: Expected Bond Excess Returns.** From the perspective of an unbiased econometrician, average optimism (pessimism) drives positive (negative) variation in the bond risk premium due to a wealth-weighted belief aggregation bias. Moreover, the model also implies that the history of disagreement (distant lags) should predict risk premia even after controlling for contemporaneous disagreement. This is because the relative wealth across agents depends on η_t, which depends on the history of disagreement Ψ_t. Indeed, previous individual decisions in bond risk exposure determine today's individual relative wealth, depending on how market shocks have been realized.

VII. Data

We use an extensive dataset on the distribution of beliefs to learn about the relative importance of the channels through which heterogeneity can affect asset prices. This section discusses the data sources and construction of variables designed to proxy for the state vector (g_t, η_t, Ψ_t).

BlueChip Financial Forecasts Indicators (BCFF) is a monthly publication providing extensive panel data on expectations by agents who are working at institutions active in financial markets. Unfortunately, at the start of this project digital copies of BCFF were available only since 2001. Thus, we obtained the complete BCFF paper archive directly from Wolters Kluwer and proceeded to digitize all the data. The digitization process required inputting around $750,000$ entries of named forecasts plus quality control checking and was completed in a joint venture with the Federal Reserve Board. The resulting dataset represents an extensive and unique dataset to investigate the role of formation of expectations about the compensation for bearing interest rate risk. Each month, BlueChip carry out surveys of professional economists from leading financial institutions and service companies regarding all maturities of the yield curve and economic fundamentals and are asked to give point forecasts at quarterly horizons out to 5-quarters ahead (6 from January 1997). While exact timings of the surveys are not published, the survey is usually conducted between the $25th$ and $27th$ of the month and mailed to subscribers within the first 5 days of the subsequent month, thus our empirical analysis is unaffected by biases induced by staleness or overlapping observations between returns and responses.

To proxy for belief dispersion, we compute a simple average of the cross-sectional inter-quartile range about 1,2,3 and 4-quarter ahead forecasts for real GDP and GDP deflator. We use these measures to study the effects and relative importance of real (Ψ_g) and inflation disagreement (Ψ_π).[34] Figure 6 plots the time series for our macroeconomic disagreement proxy along with an economic policy uncertainty factor (UnC_t) studied by Baker, Bloom, and Davis (2013).[35] Disagreement on the real economy has a significant business cycle component: in all previous three NBER economic recessions since 1990, Ψ_g is low before the recessions and it increases to peak at the end of the recessions (this occurs in 1991, 2002, and 2009). This is interesting since financial commentaries are known to report large disagreement about the state of the economy especially during recessions. We find large positive correlation between the policy uncertainty variable UnC_t and our measures of belief dispersion Ψ_g (correlation = 0.58). This is interesting since the UnC_t index assigns a weight equal to just 1/6 to forecaster disagreement about inflation. The remaining components of the index are 1/2 a broad-based news index, 1/6 a tax expiration index and 1/6 a government purchases disagreement measure. This suggests the existence of a common component in driving both policy uncertainty/disagreement and economic disagreement.

[Insert figure 6 here.]

VIII. Empirical Results

In this section, we investigate the empirical properties of the model. The first set of testable implications relates to the contemporaneous link between disagreement and the shape of the yield

[34] Dispersion in inflation expectations has recently been studied by Wright (2011), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2013), and Hong, Sraer, and Yu (2014).

[35] The economic uncertainty proxy plotted here is available for down from www.policyuncertainty.com

curve. The second set relates to the dynamics of expected bond risk premia and are studied in the context of predicting regressions.

A. Disagreement and Interest Rates: Contemporaneous Regressions

The first testable implication of the model (e.g. H_{01}) predicts that, conditional on expectations about fundamentals ($E[g_t]$), disagreement directly affects short term interest rates. Moreover, the sign of the relationship depends on the value of γ. We test this hypothesis by regressing the 3-month interest rate on contemporaneous disagreement about real GDP growth after controlling for consensus inflation and real GDP. We run regressions using both nominal and real interest rates as a dependent variable. The real rate is computed as the difference between the 3-month nominal interest rate $y_t^{\$(3m)}$ and the 3-month expected inflation obtained from the BCFF consensus series, i.e. $y_t^{r(3m)} = y_t^{\$(3m)} - E_t[\pi(t+3m)]$. Therefore, this assumes that the inflation risk premium is negligible for 3-month bonds.

$$y_t^{r(3m)} = const + \beta_{S,1}E_t[\pi] + \beta_{S,2}E_t[g] + \beta_{S,3}\Psi_t^g + \beta_{S,4}\Psi_t^\pi + \varepsilon_t^{\text{slope}} \tag{25}$$

Table V reports the results, where all right hand variables are standardised to have zero mean and unit standard deviation. The results are consistent across nominal and real short-term interest rates. The impact of real disagreement on the 3-month real short-term interest rate is statistically significant at the 1% level and the slope coefficient is negative, both in univariate and multivariate regressions. On its own, real disagreement explains 5% of the variation in the real short rate. Even after controlling for consensus expectations on gdp growth rates, disagreement is the most statistically significant explanatory variable with an R^2 equal to 11%. As we increase the tenor of the short-term rate up to 12 months, the explanatory power increases with an R^2 reaching 23%. We find similar results when using nominal short-term Treasury rates. Similar to the real rate regressions, the slope coefficient is negative and significant at the 1% level, even after controlling for expected inflation and expected GDP growth the R^2 are substantially large, ranging from 36% (3 month rate) to 60% (12 month rate).

The second testable implication of the model (e.g. H_{02}) describes how differences in beliefs affect the slope of the term structure. This implication is important since it relates to the joint properties of short term interest rates and bond risk premia, which many models find it difficult to explain. To investigate this link, we run contemporaneous regressions of the slope on real growth rate disagreement, controlling for consensus expectations of inflation, growth and disagreement about inflation.

$$Slope(t) = const + \beta_{S,1} E_t[\pi] + \beta_{S,2} E_t[g] + \beta_{S,3} \Psi_t^g + \beta_{S,4} \Psi_t^\pi + \varepsilon_t^{\text{slope}} \tag{26}$$

We measure the slope of the yield curve as the difference between the five and one year yield: $Slope_t = y_t^{(5)} - y_t^{(1)}$. Table VII reports the results. Considering real disagreement, we find that the slope coefficient of Ψ_t^g is positive and statistically significant with a t-statistics of 4.47. It is known that expectations of business cycle fluctuations are strongly linked to the slope of the term structure. Indeed, the slope of the yield curve is a popular predictor of future economic growth and inflation. Nonetheless, even after controlling for consensus expectations on future economic growth and inflation, adding real disagreement in a contemporaneous regression increases the R^2 from 14% to 21%.

The statistical significance is particularly powerful in the case of real disagreement. Indeed, when we control for both disagreement on real growth Ψ_t^g and inflation Ψ_t^π, we find that real disagreement is the most significant of the two. This is consistent with the finding that the slope of the yield curve is more strongly correlated with expectation of future economic growth than inflation.

B. Disagreement and Expected Bond Returns: Predictive Regressions

The third set of joint hypothesis relates to bond risk premia (e.g. H_{03}). Fama and Bliss (1987) shows that while forward-rate forecasts of near term yield changes are poor, at horizons above one year the predicting power of predicting regressions is remarkable. The term structure literature has studied the extent to which the shape of the yield curve forecasts future bond returns. Indeed, a formidable literature confirmed and extended this result to show that bond returns are predictable (e.g., Cochrane and Piazzesi (2005)). These results are important since interest rates and bond predictability is related to the time-varying properties of the stochastic discount factor. Thus, they provide significant clues about the fundamental drivers of the equilibrium dynamics.

The spot rate is the sum of expected inflation and an expected real return. Thus, in general, these results can be explained by the ability of forward spreads to forecasts the one year inflation rate, the real return on one year bonds, or both. The model discussed in our paper suggests that a potential explanation is the existence of a common component driving both forward spreads and expected bond return. This common factor is disagreement. The link between this disagreement and the shape of the yield curve is indeed confirmed by the results in Table VII. In this section, we consider predicting regressions in order to focus on the link between disagreement and risk premia.

We run two complementarity regressions. In the first regression, we quantify the extent to which disagreement affects 1-year interest rates by considering an augmented specification in the spirit of Fama and Bliss (1987):

$$y_{t+12}^{(1)} - y_t^{(1)} = \alpha + \beta_1(f_t^{1,2} - y_t^{(1)}) + \beta_2 CP_t + \beta_3 \Psi_t^g + \beta_4 \Psi_t^\pi + \epsilon_{t+12},$$

where the dependent variable are 1-year changes in 1-year yields and the independent variables are the 1-year forward-spot spread, the return forecasting factor (CP_t) from Cochrane and Piazzesi (2005), and differences in belief (Ψ_t^g).[36] If disagreement is a component of bond returns, than it should be significant even after controlling for the current shape of the yield curve, namely the forward-spread and the CP_t factor. Table VI summarizes the results. First, row (i) confirms that the forward spot spread *does* contain information for expected spot changes. The beta on the forward spot spread is 0.32 with a t-statistic of 1.79 and explains 10% of the variation in 1-year changes in the 1-year rate. After controlling for CP_t we find that, conditional on a steep forward curve, the Cochrane-Piazzesi factor predicts an economically large drop of short term rates over the following year with a standardized point estimate of -0.44 and a t-stat of -3.50. Rows (iii) - (vi) show the role played by disagreement. Consistent with the model prediction, we find positive shocks to disagreement predict declining short rates (negative loadings) with t-statistics of -3.53 and -2.27, depending on whether controlling for the forward spot spread or the Cochrane-Piazzesi factor.

[36] This approach closely follows Cochrane and Piazzesi (2005) who find that, not only is there a factor in the cross-section of yields that drives risk prices, but also that this factor predicts lower short rates in the future. We construct CP_t in sample using forward rates as in Cochrane and Piazzesi (2005)

These predictive regressions suggest that the information content of disagreement is remarkable. Indeed, after including Ψ_t^g, the R^2 of a Fama and Bliss (1987) regression increases from 10% to 23%; similarly, the R^2 of a Cochrane and Piazzesi (2005) regression, which includes both CP_t and the forward spread, increases from 25% to 34%.

It should be noticed that, although Ψ_t^g does not remove the statistical significance of the forward spread, β_1 increases toward 1.0, which is the value predicted by the expectation hypothesis. This suggests that Ψ_t^g may indeed capture part of the time-variation in bond risk premia, which is consistent with the model implication that an important channel of time-variation in bond risk premia is disagreement. Finally, row (vi) confirms that real disagreement is statistically more significant than disagreement about inflation.

Since the spot rate is the sum of expected inflation and an expected real return, the natural question is whether disagreement help explain bond returns because of a link between disagreement and expected real return (bond risk premia). Indeed, the third testable implication of the model is that time-variation in bond risk premia is (also) driven by (i) differences in beliefs about real growth and (ii) variations in relative wealths. The second component comes directly from equation (**??**), since in equilibrium ω^i, which is investor's i consumption share, directly depends on η_t and therefore on agents' beliefs history as discussed in the theory section above. Conditional on a sample period with a history of large belief dispersions, we should expect a large subsequent redistribution of wealth.

We conduct our tests in two stages. Let us define $hprx_{t,t+12}^{(n)}$ as the holding period excess bond returns of a n-year bond over an investment horizon of 12 months, from t to $t+12$. First, we investigate whether date t disagreement is important for explaining variation in 1-year excess returns on $n = 2 \ldots 10$-year bonds:

$$hprx_{t,t+12}^{(n)} = const + \beta_1 \Psi_t^g + \beta_2 \Psi_t^\pi + \varepsilon_{t,t+12}^{(n)} \tag{27}$$

Table VIII reports point estimates and t-statistics corrected for autocorrelation and heteroskedacity. We find that the slope coefficient of Ψ_t^g is positive and strongly significant for bond maturities up to eight years. The t-statistics of the slope coefficients on real disagreement for two and five-year bonds return is 3.19 and 2.48, respectively, with an R^2 of 16% and 10%. This is consistent with the implications of the model that suggests that real disagreement affects both the drift and local volatility (price of risk) of the stochastic discount factor. Disagreement about inflation, on the other hand, is not significant.

Second, to study whether Ψ_t^g affects bond risk premia also via its impact on the distribution of wealth (i.e. η_t) we run excess returns regressions including, at the same time, two terms: (i) contemporaneous real disagreement Ψ_t^g; and (ii) preceding monthly lags of disagreement from $h = 1, \ldots, 12$-months Ψ_{t-h}^g

$$hprx_{t,t+12}^{(n)} = const + \beta_1 \Psi_t^g + \beta_1 \Psi_{t-h}^g + \varepsilon_{t,t+12}^{(n)} \tag{28}$$

The top and bottom panels of table VIII report the results for $n = 2$ and $n = 5$ year bonds, respectively. The results show that lagged disagreement Ψ_{t-h}^g is highly significant up to $t-6$ months for the excess returns dynamics of a two year bond. Consistent with previous results, the slope coefficient is positive, as predicted by a model with $\gamma < 1$. Interestingly, both contemporaneous and lagged disagreement maintain statistical significance even when they are present at the same time. In the case of five year bonds, we find that the impact of small lag in real disagreement dominates the contemporaneous one for lags up to 6 months. This highlights the importance of long-run disagreement on the price of risk because of its role in affecting the dynamics of agents' wealth distribution.

C. Discussion of Results

When we examine the joint empirical results of all three sets of hypothesis, we find consistent evidence of the existence of a speculative channel driving, at the same time, the short term rate, the slope of the term structure and bond excess returns. The signs of the slope coefficients are all jointly consistent with economies in which the marginal trader is a financial institution with small risk aversion ($\gamma < 1$). This is consistent with a large literature that highlights the potential importance of the financial intermediation industry on capital markets. When asset under management of delegated portfolio managers becomes significant, their incentives structure may affect their effective risk aversion and their speculating activity may ultimately affect equilibrium asset prices.

Moreover, the findings in this paper show that speculation among heterogeneous low risk aversion agents is empirically consistent with several empirical regularities in bond markets which are otherwise difficult to be reconciled with single agent models with large risk aversion. Indeed, consistent with herogeneous beliefs models with $\gamma < 1$, we find that disagreement is negatively correlated with short-term interested rates. Moreover, large levels of disagreement can reduce the average short term rate to more realistic levels than those implied by homogeneous consumption-based models requiring large risk aversion (the risk free rate puzzle). At the same time, when we regress the slope of the term structure on disagreement, the slope coefficients are positive and strongly significantly. Jointly with the first, this second result also supports economies with a strong speculative motive for trading. Finally, predictive regressions tests which use holding period bond excess returns as dependent variable find that greater levels of disagreement correlates with greater expected bond excess returns. This is consistent with the implication of this class of models in which disagreement affects the stochastic discount factor so that empirical proxies help to explain the dynamic of expected excess returns.

These results are of significance in the context of three long standing questions that have been debated in an extensive stream of the asset pricing literature.

First, as Beeler and Campbell (2009) argue, many popular homogeneous agent economies can generate either pro-cyclical real short term interest rates or upward sloping real term structure, but not both at the same time. For instance, in long-run risk models a necessary condition for an upward sloping term structure is negative autocorrelation in consumption growth. However, consumption growth shows positive autocorrelation which, in these models, would imply a negative slope in the term structure of interest rates. Our empirical results suggest that a possible explanation is due to the role played by speculative channels in economies with heterogeneous agents. Indeed, in these economies larger real disagreement can help generating a steeper yield curve, even in absence of ad-hoc specifications of the inflation dynamics. This link helps to reconcile these models with the data.

Second, the results are also useful in the context of the debate about the role played by frictions in economies with disagreement. Models with short selling constraint have opposite implications, in terms of expected excess returns, than our frictionless model with speculation. If markets were subject to frictions such as short-selling constraints, larger disagreement would lead prices to reflect the beliefs of the optimist (Hong, Sraer, and Yu (2014)). Indeed, the larger the disagreement, the larger the buying pressure on long term Treasury bonds by agents who believe that future interest rates will fall. Because of short-selling constraints, pessimistic agents cannot sell short so that long-term bond prices increase, thus flattening the term structure. In this class of models, larger disagreement correlates with lower expected excess returns and lowers the slope of the term structure. In the frictionless economy calibrated earlier, on the other hand, some of these implications are reversed. If risk aversion is sufficiently low, large belief-motivated trade has the effect of increasing the slope, due to the large amount of risk shifting in equilibrium. Our empirical results are consistent with this second channel and show that disagreement can have significant impact on asset prices even in absence of short-selling constraints.

Finally, these results contribute to the debate on the relative importance of real disagreement versus inflation disagreement. Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2013) and Hong, Sraer, and Yu (2014) propose two alternative channels in which disagreement about inflation affects the average level of nominal interest rates. The first paper studies a frictionless economy with disagreement about expected inflation. In this economy nominal interest rates increase in inflation disagreement, which the authors find support for in the data. The second, on the other hand, studies the role of short-selling in bond markets and find a negative relationship in the data. Neither of these papers studies the impact on the slope of the term structure or expected excess returns. Empirically, our results are consistent with Hong, Sraer, and Yu (2014) as disagreement reduces short-term nominal interest rates. When we compare the marginal impact of different sources of disagreement, however, we find robust evidence that most of the effect is due to real disagreement. Indeed, once we control for real disagreement, the role of disagreement on inflation is not statistically significant. This is consistent with an inflation dynamics providing information about the real economy but not being a determinant of the real economy, as in Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2013).

D. Robustness

We subject our predictive regressions to a set of robustness tests that are summarised in tables X and XI.

Firstly, in the predictive regressions above we computed asymptotic standard errors using a

Newey and West (1986) correction for serial correlation because the continuously compounded annual return has an MA(12) error structure under the null hypothesis that one-period returns are unpredictable. This approach has a well documented concern that asymptotic inference generates biases in small sample. To address this concern, we employ the bootstrap approach suggested by Bekaert, Hodrick, and Marshall (1997) and Bekaert and Hodrick (2001). Bootstrapped p-values are computed by comparing Newey-West t-statistics to the distribution implied by 1000 artificial time-series generated using a stationary block bootstrap procedure with 12-month block size (Politis and Romano (1992)).

Secondly, one might be concerned that dispersion in beliefs is proxying for some alternative structural factor, such as interest rate volatility or the volatility of fundamentals. To address this concern we control for a number of factors that can be classified as (*i*) structural factors; (*ii*) volatility factors; and (*iii*) reduced form factors.

Structural Factors: From Fontaine and Garcia (2012) we take the funding liquidity factor, which is a time-varying risk factor in economies in which financial intermediaries face priced shocks to funding conditions. Vayanos and Vila (2009) and Greenwood and Vayanos (2014) study term structure models in which risk-averse arbitrageurs absorb shocks to the demand and supply for Treasury bonds. These shocks alter the price of duration risk and thus affect both bond yields and expected returns. Building on these authors work Malkhozov, Mueller, Vedolin, and Venter (2016) study a model with endogenous supply shocks predict that the outstanding quantity of mortgage-backed securities duration is positively linked to future excess bond returns. In economies with external habit preferences, such as Campbell and Cochrane (1999), time variation in risk compensation arises because of an endogenously time-varying price of risk. Shocks to the current endowment affect the wedge between consumption and habit, i.e. the consumption surplus, which induces a time-varying expected returns. We follow Wachter (2006) and use a weighted average of 10 years of monthly consumption growth rates $\sum_{j=1}^{120} \phi^j \Delta c_{t-j}$, where the weight is set to $\phi = 0.97^{1/3}$ to match the quarterly autocorrelation of the P/D ratio in the data.[37] In long-run risk economies with recursive preferences (see e.g. Bansal and Yaron (2004)), time variation in risk compensation arises from economic uncertainty (second moments) of the conditional growth rate

[37] For consumption data we obtain seasonally adjusted, real per-capita consumption of nondurables and services from the Bureau of Economic Analysis.

of fundamentals. We obtain a proxy for economic uncertainty following Bansal and Shaliastovich (2013): first, we fit a bivariate $VAR(1)$ using our survey data on consensus expectation of GDP growth and inflation. Then, we regress the sum of the squared residuals between t and $t+12$ months on time-t yields. Finally, we take the square root of the fitted values as an estimate of conditional volatility of expected real growth ($\sigma(g)$) and expected inflation ($\sigma(\pi)$).

Volatility Factors: An important stream of the literature studies the empirical performance of stochastic volatility models in terms of their ability to forecast expected future yield changes (Dai and Singleton (2000)). Motivated by this literature we consider two proxies for volatility risk. The intra-month sum of squared returns on a constant maturity 30-day Treasury bill as a proxy for short rate volatility, denoted by $\sigma_y(3m)$. The realised treasury jump risk measure first studied by Wright and Zhou (2009), which we update to include the latest sample period and denote 'Jump'.

Reduced Form Factors: We control for the Cochrane and Piazzesi (2005) return forecasting factor, which is a tent-shaped linear combination of forward rates that has been shown to contain information about future bond returns, and subsumes information contained in the level, slope and curvature of the term structure. We denote this factor CP. Finally, we update the Ludvigson and Ng (2009) dataset through 2015 but throw away any information on prices so that our panel only contains information on stationary growth rates. We control for macro predictability from the first principle component of this panel which we denote LN.

Table X and XI report estimates for 2-year and 5-year maturity bond excess returns, respectively. The first column includes disagreement about GDP and the remaining columns include controls. Summarising, for both sets of regressions we find Duration, $\sigma(g)$ and Jump factors are significant at the 5% level or lower while we do not find statistical for the remaining alternative controls. The signs on these predictors are consistent with their authors original interpretations that positive shocks drive up expected excess returns. Most important, neither of these factors are affecting the significance of Ψ_t^g whose largest bootstrapped p-value is 3%. The only exception to this finding is in table XI in specification (5) in which the p-value for Ψ_t^g is lowered to 10%, however, at the same time $\sigma(\pi)$ does not enter significantly. Interestingly, we find that the significance of both the CP and LN factors are reduced after controlling for Ψ_t^g. Finally, in specification (10) we consider a multivariate regression including Ψ_t^g, Duration, $\sigma(g)$, and Jump factors. The $\overline{R}^2$ in these regressions produces an impressive 54% and 40% on 2-year and 5-year bonds, respectively. Most of this predictable variation is coming from the Jump risk factor and, indeed, this factor is driving out some of the statistical significance of the Duration and $\sigma(g)$ factors. However, the significance of Ψ_t^g is unaffected with p-values of 2% for both maturities.

IX. Conclusion

A vast empirical literature documents that compensation for holding long dated nominal bonds in excess of short dated bonds is positive (on-average) and predictable ahead of time. The existence of such time-varying risk premia is one of the most interesting and challenging topics in fixed income and the weak empirical link between observable macro variables and bond market dynamics is a long standing puzzle.

From a theoretical perspective, the findings in this paper suggest that many properties of bond markets can be understood in terms of belief-based speculative trading. Our empirical analysis finds clear support for this channel. The first key message from this paper is that subjective beliefs and the distribution of wealth are primitive characteristics that play a first order effect in the functioning of bond markets. The second message of this paper is about the potential role that is played by agents, such as intermediaries and hedge funds, with low risk aversion in the presence of disagreement.

These results raise several interesting additional questions. First, in our model we take risk aversion as an exogenous parameter. However, propensity to bare risk may be endogenous and may depend on a variety of characteristics, such as incentives, corporate governance, market structure. It would be interesting to study structural economies in which Treasury bond prices may depend on some of these features directly (see, for related results, Buraschi, Kosowski, and Worrawat (2014)). Second, since the economy we study generates implications for equilibrium volumes of trade, it would be interesting to study the extent to which bond trading volumes are correlated to disagreement, as opposed to shocks to fundamentals. Lastly, with power utility investors, $\gamma < 1$ implies $EIS > 1$. Therefore, a natural extension to our analysis would be to separate the roles played by risk aversion versus elasticity of intertemporal substitution. We leave these questions to future research.

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X. Figures

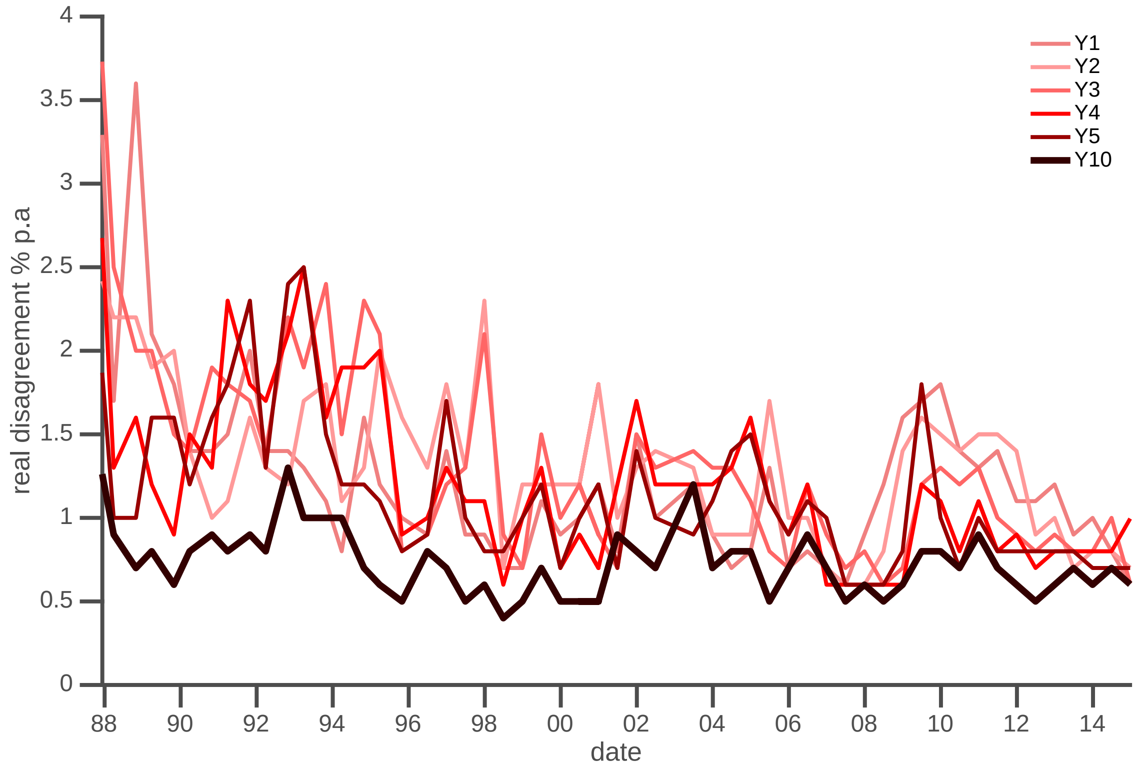


Figure 1. Term Structure of Real Disagreement
This figure plots disagreement defined as the average of the highest ten forecast minus the lowest ten forecasts for real GDP growth from the BlueChip Financial Forecasts survey. Time series is quarterly and the forecasts are averages for horizons one (Y1) to ten years (Y10).

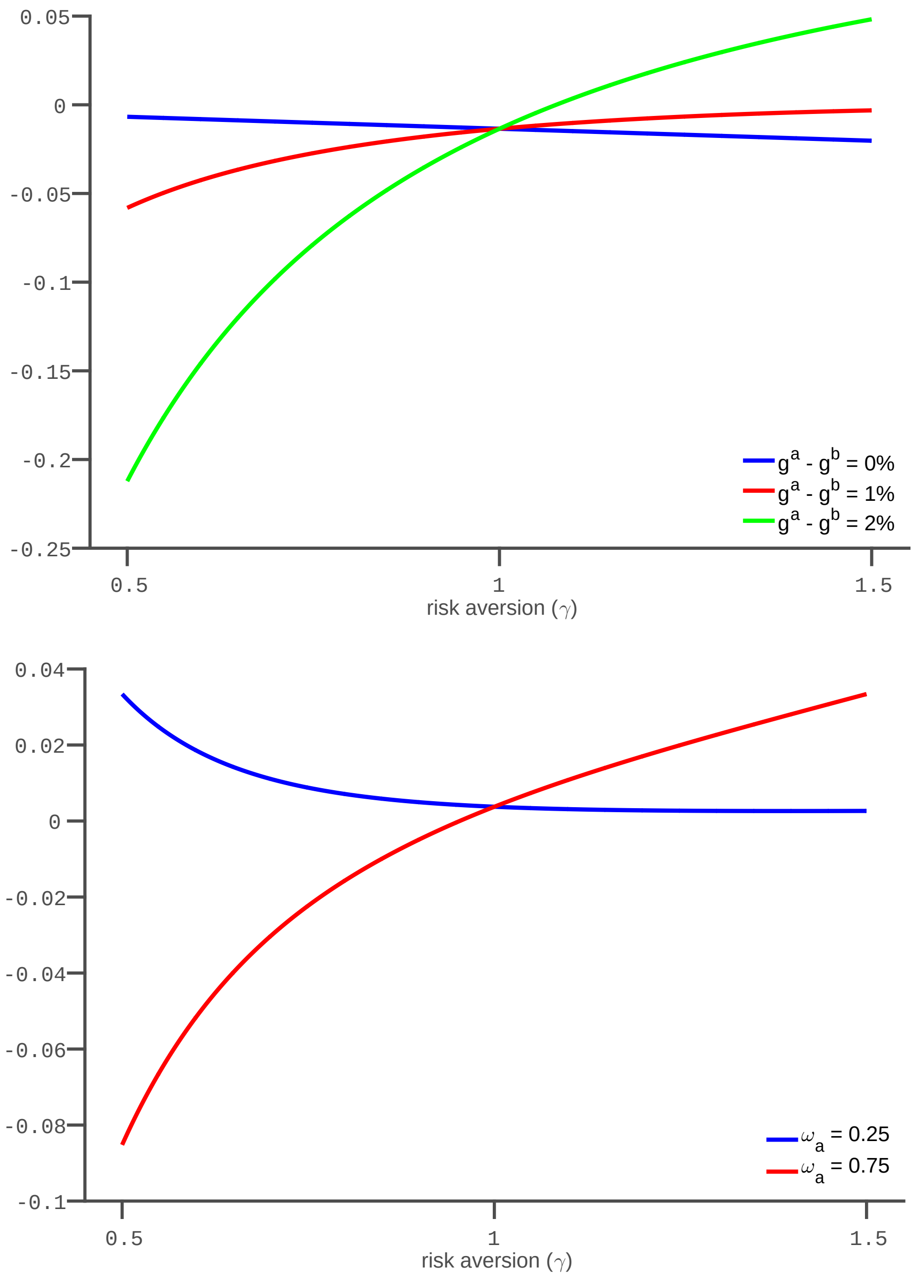
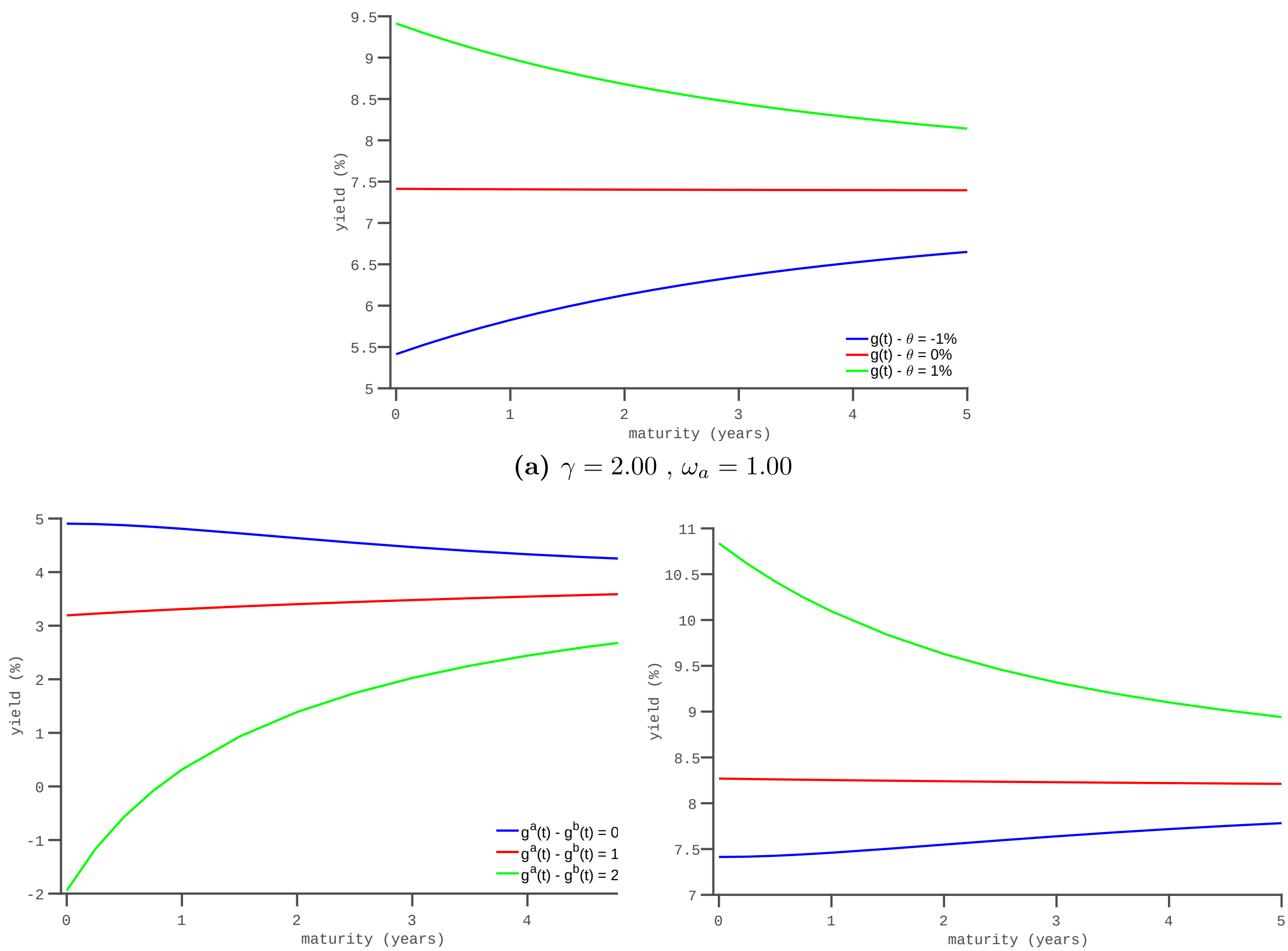


Figure 2. Short Rate Sensitivites

The top panel plots the sensitivity of the short rate with respect to disagreement as a function of risk aversion for disagreements of zero, one, and two percent. The bottom panel plots the sensitivity of the short rate with respect to the Radon-Nikodym derivative $\eta_t = \frac{\omega_b}{\omega_a}$ for optimistic ($\omega = 0.75$) and pessimistic ($\omega_a = 0.25$) economies.



(a) $\gamma = 2.00$, $\omega_a = 1.00$

(b) $\gamma = 0.50$, $\omega_a = 0.50$

(c) $\gamma = 2.00$, $\omega_a = 0.50$

Figure 3. Term Structure

This figure displays the term structure of interest rates for three economies. The top panel plots bond yields for a homogeneous economy populated by a single investor with risk aversion equal to two. The three lines plot yields for conditional consumption growth rates equal to, one percent above, and one percent below the the unconditional growth rate of the economy. The bottom two panels plot yields in heterogeneous agent economies with risk aversions one half (bottom left panel) and two (bottom right panel). In the heterogeneous agent economy plots agents have equal wealth and we plot disagreement between 0% and 2% as a mean preserving spread around the unconditional mean growth rate.

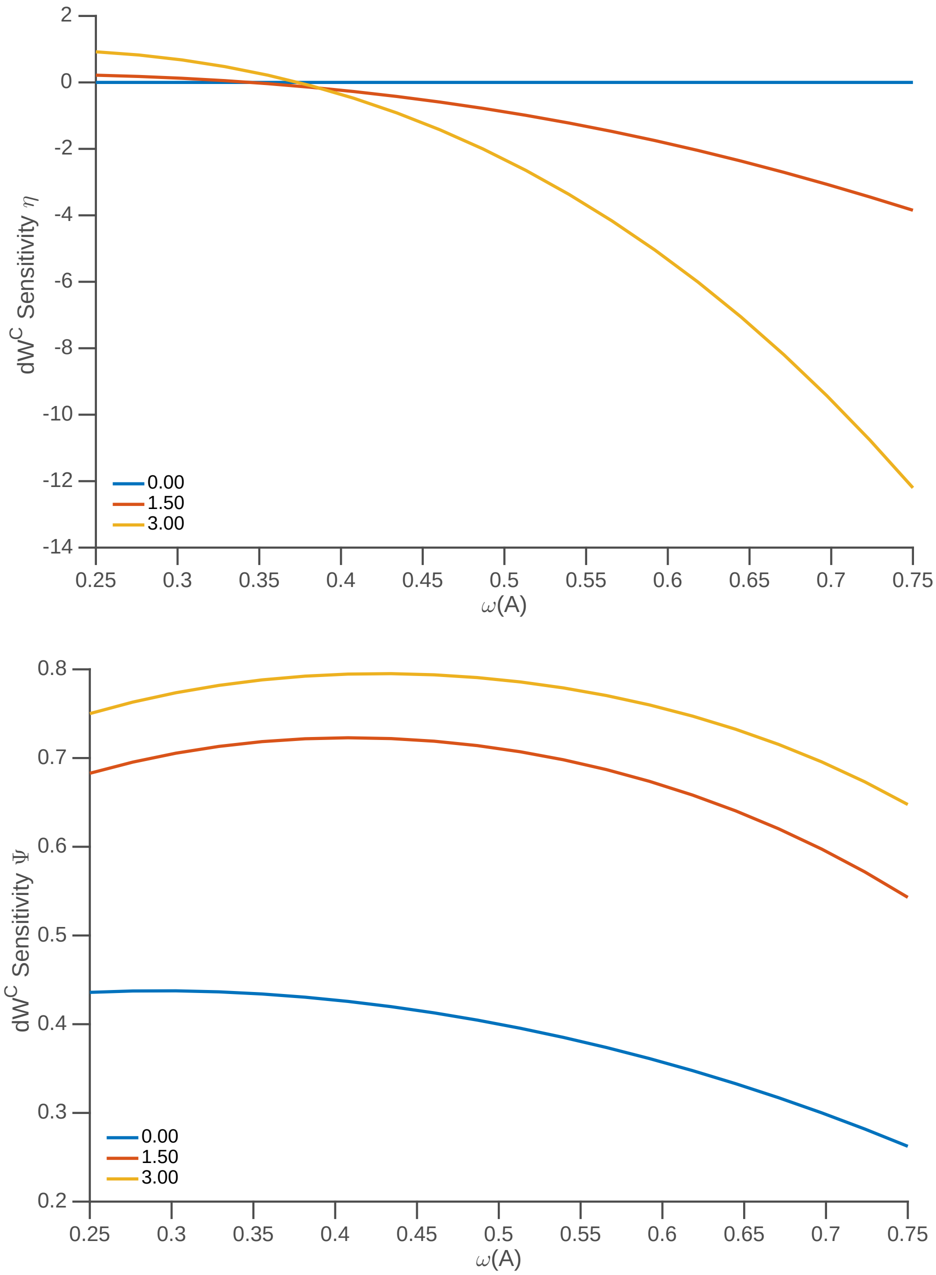


Figure 4. Bond Return Sensitivities to Consumption Shocks.
The sensitivity of bond returns to dW_c shocks is given by the sum of three terms:

$$\sigma^{\tau}_{b,c} = \frac{1}{B^{\tau}} \left[\frac{\partial B^{\tau}}{\partial \widehat{g}^{a}} \, \sigma_{c,g} + \frac{\partial B^{\tau}}{\partial \eta} \, (-\Psi_t) + \frac{\partial B^{\tau}}{\partial \psi} \sigma_{c,\Psi} \right]$$

For the a bond with maturity $\tau = 5$ years and for $\gamma = 0.75$, panel (a) plot the component of this gradient due to η (the middle term) while panel (b) plots the component of this gradient due to Ψ (the last term). Sensitivities are plotted as a function of the relative wealth of agent a (along the x-axis) and for three different levels of disagreement (Ψ).

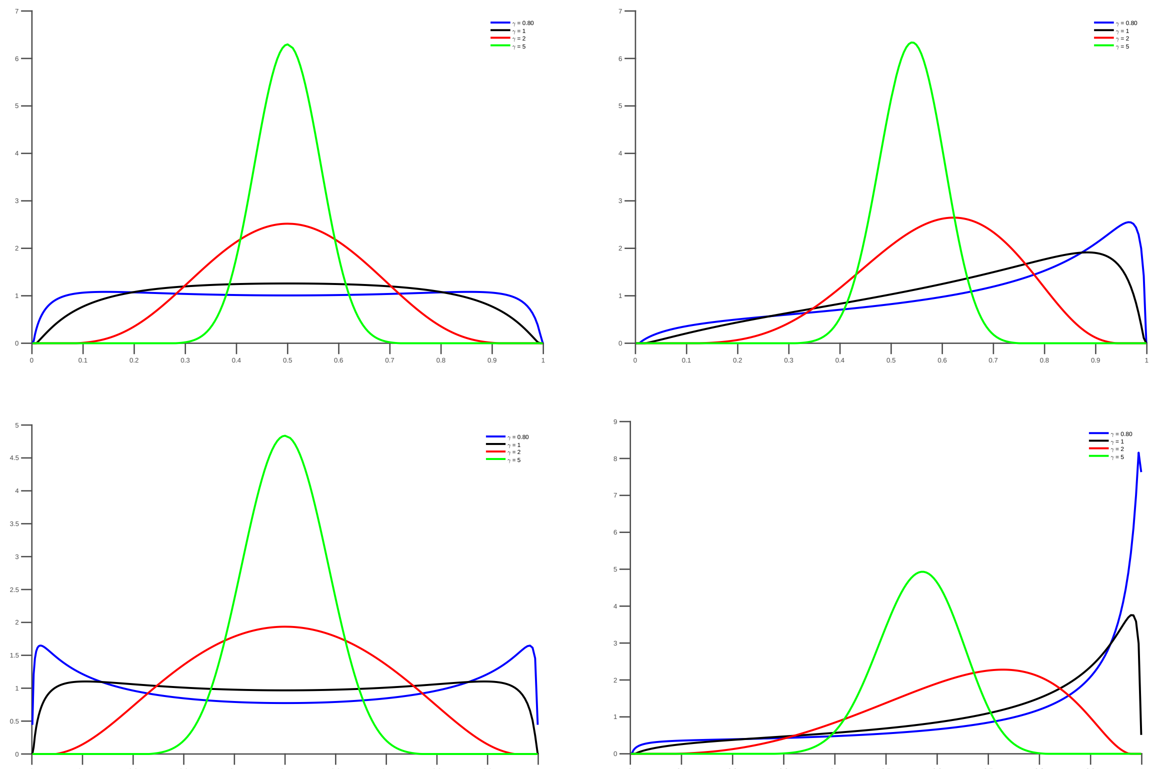


Figure 5. The Distribution of Wealth.
Left panels plot the wealth distribution when both agents hold equally incorrect beliefs. The right panels of figure plot wealth distributions when one agent holds the correct belief and the other agent holds the incorrect belief. Top panels show wealth distributions after 25 years and bottom panels plot the wealth distributions after 50 years. Each panel considers level of risk aversion equal to $\{0.80, 1.0, 2.0, 5.0\}$

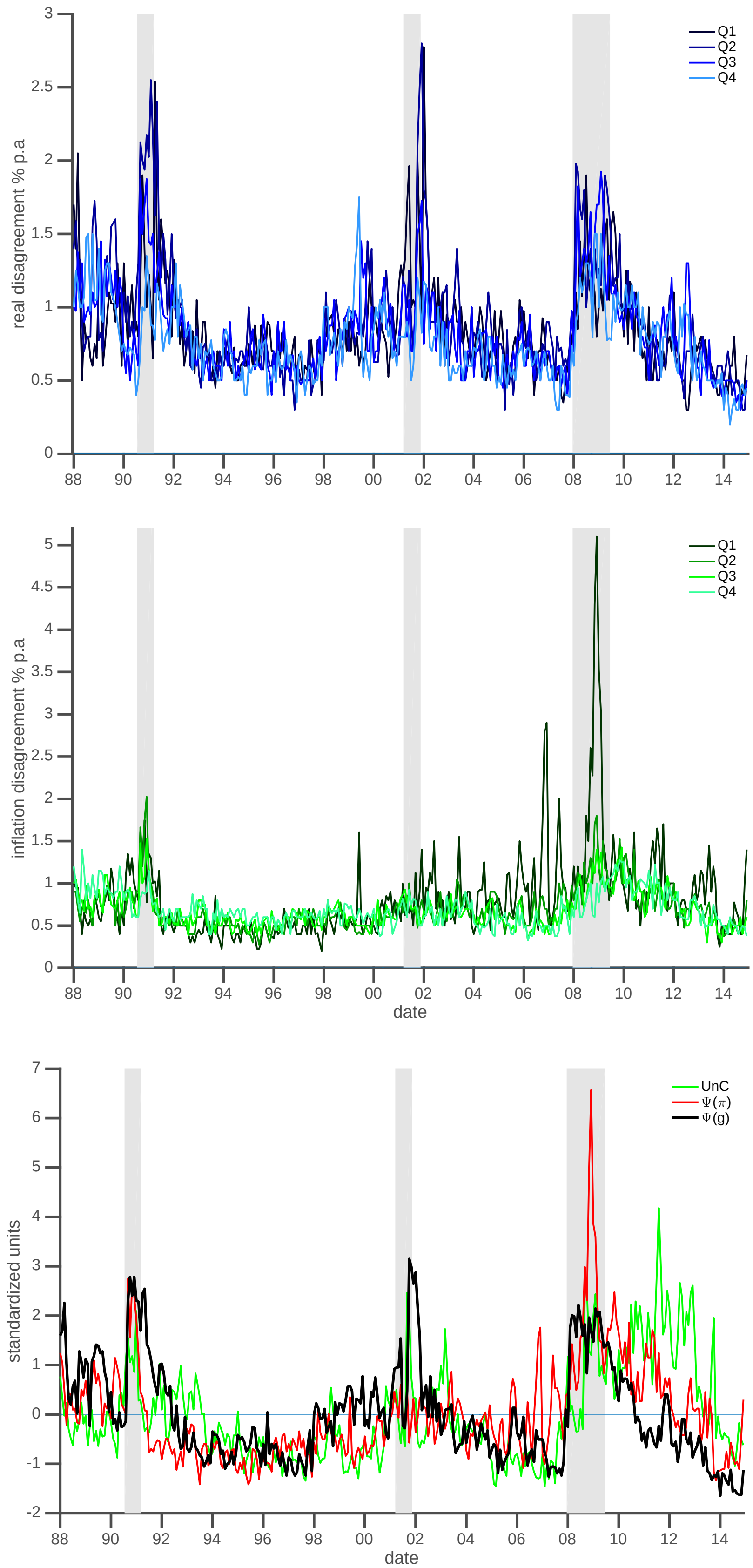


Figure 6. Disagreement Factors
This figure displays time-series plots for 1-quarter to 4-quarters ahead disagreement about GDP growth (left panel) and inflation (right panel).

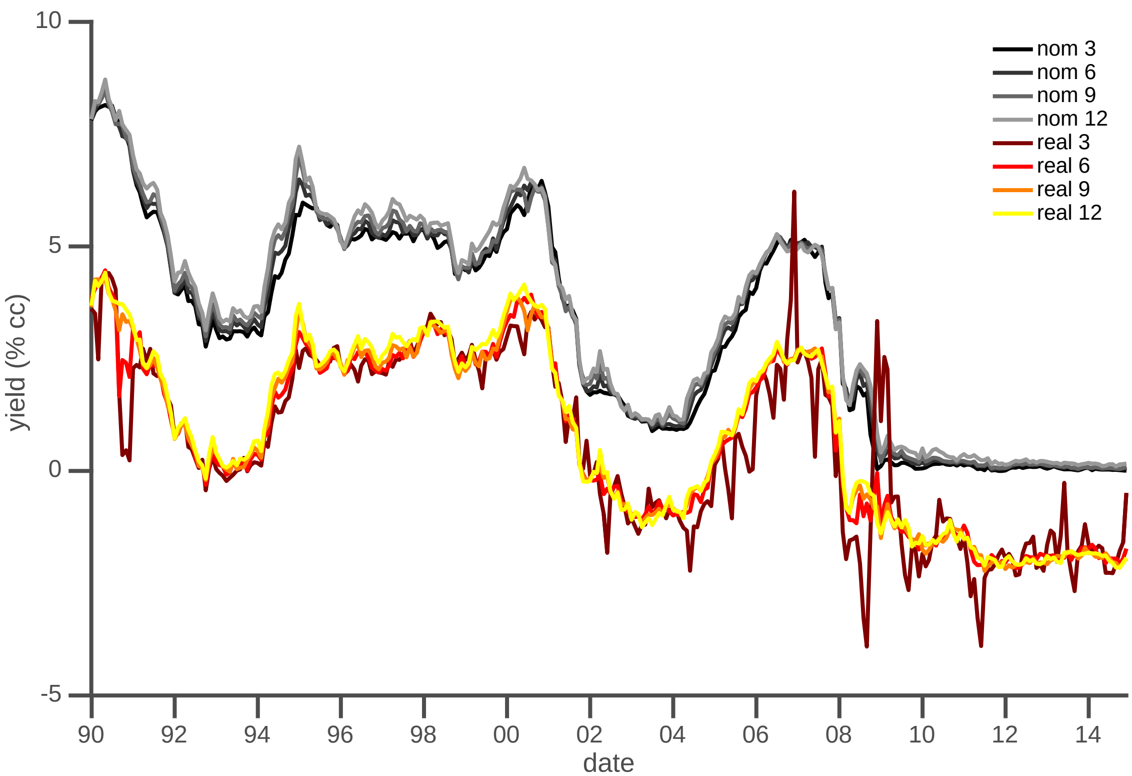


Figure 7. Real and Nominal Short Rates

This figure plots time series of real and nominal short rates. Nominal rates are plotted for 3, 6, 9 and 12-month maturities. Real short rates are proxied by subtracting 3, 6, 9 and 12-month expected inflation from the BlueChip Financial Forecasts survey from the corresponding nominal rate.

XI. Appendix C: Tables

Table I. Calibrated Parameters

This table summarises the calibrated parameters describing the state space perceived by each agent.

Common	δ	σ_c	κ_g	σ_g	θ	π	σ_q
Parameters	$-\log(0.99)$	0.0116	0.2424	0.01115	1.96%	3.54%	0.0121
Subjective	θ^a	θ^b	$\rho^a_{c,g}$	$\rho^b_{c,g}$	$\rho^a_{q,g}$	$\rho^b_{q,g}$	
Parameters	$\theta + 0.50\%$	$\theta - 0.50\%$	0.50	-0.50	0.50	-0.50	

Table II. Calibrated versus Empirical Consumption Process

Table summarises data and model implied moments for annual and monthly real per capita consumption growth of non-durables plus services. Data from the U.S. Bureau of Economic Analysis (BEA). The dynamics for consumption are given by

$$\begin{aligned} dC_t/C_t &= g_t dt + \sigma_c dW_t^c, \\ dg_t &= \kappa_g\big(\theta - g_t\big)dt + \sigma_g dW_t^g, \\ 0 &= \langle dW_t^c, dW_t^g \rangle \end{aligned}$$

	Annual		Monthly	
	Data	Model	Data	Model
$E\left[\ln \frac{C(t+\tau)}{C(t)}\right]$	1.9686	1.9547	1.9609	1.9547
$\sigma\left[\ln \frac{C(t+\tau)}{C(t)}\right]$	1.2863	1.6043	1.1587	1.1589
AR(1)	0.4985	0.4057	-0.1697	0.0729
AR(2)	0.1978	0.3184	0.0510	0.0715

Table III. Quantitive Predictions I

Table summarises data and model implied moments for the level ($\frac{1}{5}\sum_{n=1}^{5} y^n$) of the nominal term structure, the slope ($(y^{(5)} - y^{(1)})$) of the term structure, the 3-month rate ($y^{(0.25)}$), the volatility of yield changes for one year ($\sigma^{(1)}$) and five year ($\sigma^{(2)}$) and bond risk premia on two and five year bonds ($hprx^{(2,5)}$).

	Level	Slope	$y^{(0.25)}$	$\sigma^{(1)}$	$\sigma^{(5)}$	$hprx^{(2)}$	$hprx^{(5)}$
	PANEL A: $\theta_a - \theta_b = 1\%$, $\rho^a_{c,g} - \rho^b_{c,g} = 1$, $\rho^a_{c,q} - \rho^b_{c,q} = 1$						
$\widehat{g}^a - \widehat{g}^b$	$\omega_a = 50\%$, $\gamma = 0.75$						
0.00%	4.24	-0.45	5.90	0.50	0.28	-0.01	-0.01
0.50%	3.94	0.09	4.90	1.97	0.41	0.00	-0.01
1.00%	3.27	1.39	1.91	3.28	0.59	0.00	-0.02
1.50%	2.33	3.27	-3.08	4.12	0.77	-0.01	-0.02
	$\omega_a = 50\%$, $\gamma = 2.00$						
0.00%	10.80	1.24	8.41	1.48	0.89	-0.06	-0.10
0.50%	11.26	0.59	9.58	3.52	1.26	-0.08	-0.13
1.00%	12.33	-1.09	13.07	5.82	1.73	-0.10	-0.16
1.50%	13.98	-3.77	18.89	8.05	2.25	-0.13	-0.20
	$\omega_a = 75\%$, $\gamma = 0.75$						
0.00%	4.65	-0.39	5.90	0.43	0.30	-0.01	-0.01
0.50%	4.47	-0.03	5.23	1.74	1.07	0.24	0.53
1.00%	4.00	0.93	3.08	3.85	2.05	1.11	2.10
1.50%	3.29	2.38	-0.58	7.32	3.25	3.12	5.10
	$\omega_a = 75\%$, $\gamma = 2.00$						
0.00%	10.29	1.04	8.41	1.74	1.08	-0.06	-0.11
0.50%	10.83	0.48	9.53	2.65	1.14	-0.03	-0.33
1.00%	11.83	-0.85	12.40	4.96	2.93	-1.04	-2.60
1.50%	13.26	-2.92	17.02	11.33	5.41	-4.45	-7.83
	$\omega_a = 25\%$, $\gamma = 0.75$						
0.00%	4.49	-0.49	5.90	0.56	0.30	0.00	-0.01
0.50%	4.17	-0.01	5.06	1.82	0.35	0.01	-0.05
1.00%	3.55	1.09	2.73	3.02	0.48	0.18	0.08
1.50%	2.71	2.67	-1.10	3.98	0.61	0.54	0.38
	$\omega_a = 25\%$, $\gamma = 2.00$						
0.00%	10.04	0.69	8.41	1.23	0.79	-0.05	-0.09
0.50%	10.19	0.27	9.03	2.35	0.89	-0.36	-0.58
1.00%	10.77	-0.93	11.40	3.93	1.08	-0.81	-1.21
1.50%	11.76	-2.86	15.52	5.46	1.30	-1.39	-1.99

Table IV. Quantitive Predictions II

Table summarises data and model implied moments bond risk premium on two and five year bonds ($hprx^{(2,5)}$) in four cases (Panel A ... Panel D). Panel A and Panel B present model implied numbers for $\omega_a = 75\%$, $\gamma = 0.75$ and and for $\rho^a - \rho^b = 1$ and $\rho^a - \rho^b = -1$, respectively. Panel C and Panel D present model implied numbers for $\omega_a = 25\%$, $\gamma = 0.75$ and and for $\rho^a - \rho^b = 1$ and $\rho^a - \rho^b = -1$, respectively.

	$hprx^{(2)}$	$hprx^{(5)}$	$hprx^{(2)}$	$hprx^{(5)}$
	Panel A		Panel B	
0.00%	-0.01	-0.01	-0.01	-0.01
0.50%	0.24	0.53	0.31	0.54
1.00%	1.11	2.10	1.31	2.16
1.50%	3.12	5.10	3.43	5.17
	Panel C		Panel D	
0.00%	0.00	-0.01	-0.01	-0.01
0.50%	0.01	-0.05	-0.11	-0.15
1.00%	0.18	0.08	-0.17	-0.19
1.50%	0.54	0.38	-0.08	-0.08

Table V. Short Rate Regressions

OLS projections of real and nominal interest rates for maturities $n = 3, 6, 9, 12$-months on real disagreement (Ψ^g), inflation disagreement (Ψ^π), and consensus expectations about n-month GDP growth and inflation ($E[g]$, $E[\pi]$). Both left and right hand variables are standardized. t-statistics are corrected for autocorrelation and heteroskedasticity. Sample Period: 1990.1 - 2011.1

	$E[\pi]$	$E[g]$	Ψ^g	Ψ^π	$\overline{R}^2$
			3 Month Rates		
real		−0.35	−0.47		11%
		(−2.20)	(−2.92)		
nom	0.58	−0.31	−0.40		36%
	(5.80)	(−2.16)	(−2.96)		
nom	0.52	−0.41	−0.30	−0.31	42%
	(4.45)	(−3.12)	(−2.33)	(−2.79)	
			6 Month Rates		
real		−0.32	−0.44		12%
		(−1.61)	(−3.25)		
nom	0.69	−0.18	−0.30		57%
	(9.21)	(−1.81)	(−3.42)		
nom	0.61	−0.31	−0.23	−0.28	60%
	(7.77)	(−3.15)	(−2.54)	(−3.24)	
			9 Month Rates		
real		−0.35	−0.36		16%
		(−2.24)	(−2.88)		
nom	0.69	−0.20	−0.23		60%
	(8.56)	(−2.18)	(−3.51)		
nom	0.61	−0.30	−0.12	−0.27	64%
	(6.99)	(−3.86)	(−1.53)	(−3.28)	
			12 Month Rates		
real		−0.43	−0.28		23%
		(−3.34)	(−2.00)		
nom	0.64	−0.26	−0.20		60%
	(7.41)	(−2.88)	(−2.77)		
nom	0.57	−0.33	−0.06	−0.27	64%
	(6.23)	(−4.47)	(−0.72)	(−3.25)	

Table VI. 'Complementarity' Fama-Bliss Regressions

Forecasting regression of 1-year changes in the 1-year yield on the $(1, 2)$ forward spot spread ($fs^{1,2}$), the Cochrane-Piazzesi factor (CP_t), real disagreement (Ψ^g), and inflation disagreement (Ψ^π). t-statistics are corrected for autocorrelation and heteroskedasticity. Both left and right hand variables are standardized. Sample Period: 1990.1 - 2014.6

	$fs^{1,2}$	CP_t	Ψ^g	Ψ^π	$\overline{R}^2$
(i)	0.32				10%
	(1.79)				
(ii)	0.51	−0.44			25%
	(2.65)	(−3.50)			
(iii)	0.38		−0.37		23%
	(2.28)		(−3.53)		
(iv)		−0.15	−0.27		11%
		(−1.39)	(−2.27)		
(v)	0.54	−0.38	−0.30		34%
	(3.04)	(−3.53)	(−3.53)		
(vi)	0.54	−0.38	−0.31	0.01	34%
	(2.98)	(−3.54)	(−2.79)	(0.06)	

Table VII. The Slope of The Term Structure

OLS regression of the $Slope_t = y_t^{(5)} - y_t^{(1)}$ of the term structure on real disagreement (Ψ^g), inflation disagreement (Ψ^π), and consensus expectations about 4-quarter GDP growth and inflation ($E[g]$, $E[\pi]$). t-statistics are corrected for autocorrelation and heteroskedasticity. Both left and right hand variables are standardized. Sample Period: 1990.1 - 2014.6

	$E[\pi]$	$E[g]$	Ψ^g	Ψ^π	$\overline{R}^2$
Slope	0.07	0.39			0.14
	(0.98)	(5.33)			
Slope	0.06	0.41	0.25		0.21
	(0.95)	(6.35)	(4.47)		
Slope	0.10	0.44	0.19	0.12	0.21
	(1.39)	(6.70)	(2.57)	(1.54)	

Table VIII. Return Predictability Regressions

This table reports estimates from OLS regressions of annual ($t \rightarrow t+12$) excess returns on $2, 3, 4, 5$-year zero-coupon bonds on real disagreement (Ψ^g_{t-L}) and inflation disagreement (Ψ^π_{t-L}). Panels reports specifications in which the right hand variables are lagged by L periods. t-statistics are corrected for autocorrelation and heteroskedasticity. Both left and right hand variables are standardized. Sample Period: 1990.1 - 2014.6

	$rx(2)$	$rx(3)$	$rx(4)$	$rx(5)$
		Zero Lag		
Ψ^g_t	0.43	0.39	0.35	0.29
	(3.19)	(3.16)	(2.89)	(2.48)
Ψ^π_t	-0.05	-0.01	0.01	0.06
	(-0.45)	(-0.05)	(0.06)	(0.42)
$\overline{R}^2$	16%	14%	12%	10%
		3-Month Lag		
Ψ^g_{t-3}	0.51	0.48	0.45	0.40
	(3.79)	(3.56)	(3.34)	(2.97)
Ψ^π_{t-3}	-0.10	-0.05	-0.05	-0.02
	(-0.89)	(-0.44)	(-0.34)	(-0.12)
$\overline{R}^2$	21%	20%	17%	14%
		6-Month Lag		
Ψ^g_{t-6}	0.44	0.42	0.39	0.35
	(3.34)	(3.26)	(2.95)	(2.68)
Ψ^π_{t-6}	-0.08	-0.04	-0.01	0.02
	(-0.78)	(-0.37)	(-0.08)	(0.22)
$\overline{R}^2$	16%	15%	14%	12%
		12-Month Lag		
Ψ^g_{t-12}	0.29	0.28	0.26	0.24
	(2.03)	(2.06)	(1.96)	(1.81)
Ψ^π_{t-12}	-0.02	0.04	0.09	0.13
	(-0.20)	(0.32)	(0.83)	(1.22)
$\overline{R}^2$	7%	8%	9%	9%

Table IX. State Dependent Return Predictability Regressions

This table reports estimates from OLS regressions of annual ($t \to t+12$) excess returns on $2, 3, 4, 5$-year zero-coupon bonds on date t real disagreement (Ψ_t^g) and h-month lagged real disagreement (Ψ_{t-h}^g). t-statistics are corrected for autocorrelation and heteroskedasticity. Both left and right hand variables are standardized. Sample Period: 1990.1 - 2014.6

$h =$	$1m$	$2m$	$3m$	$4m$	$5m$	$6m$	$7m$	$8m$	$9m$	$10m$	$11m$	$12m$
						2-year Bond Excess Returns						
Ψ_t^g	0.15	0.12	0.15	0.19	0.23	0.27	0.30	0.33	0.34	0.34	0.34	0.34
	(1.00)	(1.32)	(1.59)	(1.97)	(2.54)	(2.91)	(3.27)	(3.54)	(3.61)	(3.48)	(3.29)	(3.18)
Ψ_{t-h}^g	0.28	0.35	0.35	0.33	0.28	0.25	0.19	0.16	0.14	0.15	0.17	0.16
	(2.67)	(3.03)	(3.01)	(2.83)	(2.50)	(2.25)	(1.72)	(1.46)	(1.29)	(1.37)	(1.59)	(1.60)
$\overline{R}^2$	18%	20%	22%	22%	21%	21%	19%	19%	18%	18%	18%	17%
						5-year Bond Excess Returns						
Ψ_t^g	0.11	0.07	0.07	0.09	0.14	0.17	0.20	0.22	0.23	0.24	0.23	0.23
	(1.52)	(0.79)	(0.73)	(0.95)	(1.48)	(1.93)	(2.27)	(2.49)	(2.59)	(2.53)	(2.40)	(2.30)
Ψ_{t-h}^g	0.23	0.30	0.34	0.34	0.30	0.27	0.22	0.20	0.19	0.19	0.21	0.23
	(2.15)	(2.50)	(2.80)	(2.82)	(2.58)	(2.51)	(2.14)	(2.08)	(1.83)	(1.82)	(2.09)	(2.34)
$\overline{R}^2$	11%	13%	15%	16%	16%	15%	13%	13%	13%	13%	13%	13%

Table X. Robustness: 2-year hprx

Table reports point estimates and p-values from regressions of annual excess returns on 2-year bond on real disagreement and a set of controls. These controls are outlined in the main body of the paper. p-values reported in brackets are computed by comparing 12-lag Newey-West t-statistics to the distribution of bootstrapped statistics implied by a stationary block resampling. Both left and right hand variables are standardized. Sample Period: 1990.1 - 2014.6

	Ψ_t^g	Liquidity	Duration	Surplus	$\sigma(g)$	$\sigma(\pi)$	Jump	$\sigma(3m)$	LN	CP	$\overline{R}^2$
(1)	0.37	-0.17									18%
	[0.01]	[0.15]									
(2)	0.45		0.33								26%
	[0.00]		[0.00]								
(3)	0.40			-0.01							16%
	[0.00]			[0.94]							
(4)	0.32				0.41						32%
	[0.00]				[0.01]						
(5)	0.29					0.29					23%
	[0.01]					[0.40]					
(6)	0.27						0.54				43%
	[0.03]						[0.00]				
(7)	0.38							0.18			19%
	[0.01]							[0.03]			
(8)	0.34								0.10		16%
	[0.04]								[0.62]		
(9)	0.35									0.18	18%
	[0.01]									[0.27]	
(10)	0.24		0.09		0.30		0.46				54%
	[0.02]		[0.41]		[0.05]		[0.00]				

Table XI. Robustness: 5-year hprx

Table reports point estimates and p-values from regressions of annual excess returns on 5-year bond on real disagreement and a set of controls. These controls are outlined in the main body of the paper. p-values reported in brackets are computed by comparing 12-lag Newey-West t-statistics to the distribution of bootstrapped statistics implied by a stationary block resampling. Both left and right hand variables are standardized. Sample Period: 1990.1 - 2014.6

	Ψ_t^g	Liquidity	Duration	Surplus	$\sigma(g)$	$\sigma(\pi)$	Jump	$\sigma(3m)$	LN	CP	$\overline{R}^2$
(1)	0.29	-0.17									12%
	[0.00]	[0.17]									
(2)	0.37		0.41								26%
	[0.00]		[0.00]								
(3)	0.28			-0.12							11%
	[0.04]			[0.46]							
(4)	0.27				0.26						16%
	[0.01]				[0.05]						
(5)	0.19					0.35					20%
	[0.10]					[0.34]					
(6)	0.19						0.49				32%
	[0.03]						[0.00]				
(7)	0.30							0.10			10%
	[0.01]							[0.15]			
(8)	0.34								-0.03		9%
	[0.02]								[0.83]		
(9)	0.27									0.19	13%
	[0.01]									[0.21]	
(10)	0.23		0.26		0.10		0.40				40%
	[0.02]		[0.01]		[0.43]		[0.00]				