

Université Paris IX Dauphine

Revisiting Ohlson's Equity Valuation Model

Frédéric Parienté

CEREG

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Abstract

The Ohlson model (OM) builds on the accounting-based residual income valuation (RIV) model for equity valuation then specifies a linear information model (LIM) for the time-series behavior of residual income. As a result, stock price is related through a simple linear formula to current book value, current profitability and future profitability. Despite its common theoretical ground with the widely used discounted cash flow (DCF) method, there was a perception among empiricists that OM performed better than DCF when implemented over a finite horizon. In this work, we show that this claim is unfounded and that both OM and RIV yield similar results to DCF when implemented correctly. Specifically we first review three common mistakes made in empirical studies supporting the claim, which led to model the steady-state continuation period in a non-consistent fashion across valuation methods. Second, we describe how the Gordon-Shapiro formula can be rewritten as a linear information model. Still we believe in the superiority of OM which consists in the formalization of LIM. It provides a framework to better capture the value drivers which should eventually yield better results in derivative OM models. The research should now focus on its main challenges of determining the value drivers and forecasting earnings.

Le modèle d'Ohlson étend la méthode comptable d'évaluation par les bénéfices anormaux en y associant un modèle d'information linéaire. La valeur de marché d'une société est alors donnée par une fonction linéaire de sa valeur comptable, profitabilité actuelle et profitabilité future. Bien qu'il dérive du même cadre théorique que la méthode des flux actualisés, il existe dans la recherche empirique la perception que le modèle d'Ohlson est meilleur que celui des flux actualisés lorsque ces méthodes sont mises en œuvre sur un horizon fini. Dans ce mémoire, nous démontrons que cette thèse n'est pas fondée et que les méthodes comptables aboutissent à des résultats équivalents à ceux des techniques financières quand elles sont correctement appliquées. Dans un premier temps, nous montrons que les études qui appuient cette thèse pâtissent de trois erreurs, qui amènent à modéliser le comportement stationnaire dans la période terminale de façon non cohérente d'une méthode à l'autre. Ensuite, nous montrons comment le modèle de Gordon-Shapiro peut être vu comme un modèle d'information linéaire. Nous reconnaissons cependant une supériorité au modèle d'Ohlson qui réside dans la formalisation du modèle d'information. Le modèle d'Ohlson offre un cadre pour une meilleure prise en compte des variables explicatives de la valeur, ce qui conduira à des modèles dérivés plus performants. La recherche peut désormais se concentrer sur sa tâche essentielle d'identification des variables explicatives et de prévision des résultats comptables.

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1 Introduction

During most of the twentieth century, the financial community has been denying to accounting data any relevance in equity valuation on the premises that accounting is based upon (historical) cost. The common belief in corporate finance is well summarized in the following statement from Appleyard (1980): “it is well known that (conventionally measured) accounting income cannot be related to a firm's capital stock in a simple way.” Again recently, one can read in Cañibano (2000): “a sign of the loss of relevance of accounting information is the increasing gap between the market value and the book value of equity of companies in financial markets”. However, for as long as the financial community has been denying accountants any say in equity valuation, accountants have been publishing promising results on accounting-based valuation models.

In 1936, Preinreich decomposed accounting earnings into *interest on investment*, a mere payment for time, and *excess income*, which is discounted to obtain the firm's goodwill. For a firm liquidated after 10 years, he plotted the evolution of the capital's book value, to which earnings are added and from which dividends are subtracted at the end of each period to compute the book value of the next period¹, and graphically showed that the firm's market value, i.e. the sum of discounted dividends, equals the initial corporate capital plus the sum of discounted excess earnings.

In 1961, Edwards and Bell, two economists, came forward with the idea that accounting needed to be restructured so that the information is compatible with present value analysis, which itself would be reformatted to output periodic income instead of a single overall project income. In economics terms, standard accounting income includes both *normal profits*, i.e. the profits that would be earned from holding a diversified portfolio with the same level of systematic risk, and *monopoly profits*, i.e. the profits in excess of normal profits. For each period, Edwards and Bell define the *total economic income* as the net investment in the project at the start of the period times the internal rate of return and, the *excess economic income* (the monopoly profits) as the total economic income of the period less the net investment in the project at the start of the period multiplied by the cost of opportunity. In Edwards and Bell's economic accounting, excess income will be used as a measure of operating income. As a result, the discounted operating income adds up to total the project's net present value.

In 1982, Peasnell showed that, not only income computed under economic depreciation but any accounting measure of income can be discounted (and adjusted) to match the firm's value given by discounting free cash flows. Similarly to his predecessors, he builds on the clean surplus relation to

¹ This relation between book value, earnings and dividends is known as the *clean surplus relation*.

equal the excess net present value to the sum of discounted excess income plus an error. From there, he expresses the (constant) economic rate of return as a weighted average of the accounting rates of return, an expression which can be further simplified under the typical steady-state assumption of a constant rate of growth for the firm's assets. Because these relations were obtained without any assumption on the type of accounting depreciation used, Peasnell rejects the claim that accounting data are distorted and cannot relate to value.

Until 1995 though, these accounting-based equity valuation models, collectively known today as *residual income valuation*, were still not getting any echo in the financial community. In Kaplan and Ruback (1995) for instance, a major milestone in empirical research on valuation models, the authors document the implementation and accuracy of the discounted cash flow model then compare its performance to valuation approaches based on multiples, which are considered as the sole alternative. Accounting-based models are not even mentioned in the study.

The publication of Ohlson (1995) created an electroshock in the research community². In the line of research of Preinreich (1936), Edwards and Bell (1961), and Peasnell (1982), Ohlson develops an equity valuation model that relates the current stock value to current accounting data. When implemented over a finite horizon, the more traditional discounted cash flow method involves the computation of a terminal value, which can represent a large percentage of the value. Because Ohlson's model uses current accounting data and because residual income, as opposed to free cash flows, does account for future profit of an investment, it created the perception that these accounting-based models brought the future forward, thus relying less on the terminal value and performing better than the discounted cash flow method. In the following years, besides a few normative studies³, many empirical studies⁴ on valuation models were published with the main interest being the comparison of the discounted cash flow method and the residual income valuation, the Ohlson (1995) model or its derivative models⁵. All publications concluded on the superiority of accounting-based approaches, taking them out of the anonymity they had known for most of the century. Finally Ohlson (1995) did not only put back accounting-based models back in fashion in equity valuation but complementary lines of research quickly adopted these models as well⁶.

As the model gained in popularity, a few researchers⁷ though started to question the enthusiasm

2 See Dessertine (2001) for a literature review on the impact of the Ohlson's model.

3 e.g. Bernard (1995), Penman (1997).

4 e.g. Penman and Sougiannis (1998), Barth et al. (1999), Dechow et al. (1999), Myers (1999), Francis et al. (2000), Courteau et al. (2001).

5 e.g. Feltham and Ohlson (1995), Feltham and Ohlson (1999), Ohlson (1999), Begley and Feltham (2002).

6 See Gehardt et al. (2001) on cost of capital and Ballester et al. (2000) on intangible assets.

7 e.g. Lo and Lys (2000), Lundholm and O'Keefe (2001).

around the Ohlson model on the premises that there was no theoretical ground to claim the superiority of accounting-based models over traditional financial approaches. So, is Ohlson (1995) indeed the revolution Bernard (1995) claimed it was? Is it performing better than traditional financial approaches? In this work, we revisit the Ohlson model to answer whether it is innovative with respect to past models and whether it performs better than discounted cash flow techniques, as claimed in many empirical studies.

This work contributes to the literature in three ways. First, it compiles most criticisms made in various places on the Ohlson model in a single document and in the focused perspective of its performance comparison to the discounted cash flow method. Second, it aims to provide more economic and accounting insights to each criticism. Third, it positions the Ohlson model not as an accomplished equity model in the continuity of past equity valuation models but rather as a framework from which (hopefully superior) valuation models will be derived. In that sense, we view the Ohlson model as the starting point of a new line of research. We finally present a series of possible derivative model, some of which are new to the literature.

The remainder of this work is organized as follows. Section 2 presents the Ohlson (1995) equity valuation model and the traditional accounting-based model it builds upon, the residual income model. Section 3 demonstrates the strict equivalence between the discounted cash flow and residual income models. Section 4 describes how to ensure the equivalence when implementing the models over finite horizon. Section 5 demonstrates that the Ohlson model does not in fact introduce any information that makes it superior to traditional models but shows how it can be used as a framework to introduce and model such information. Section 6 concludes.

2 The Ohlson model

In this section, we describe the equity valuation model presented in Ohlson (1995).

2.1 Present value

Under the neo-classical multi-period framework (Fisher 1930), the market value of a firm's equity $P(t)$ at year t equals the present value of expected dividends $d(t)$ discounted at a constant factor R :

$$P(t) = \sum_{\tau=1}^{\infty} \frac{E[d(t+\tau)]}{(1+R)^\tau} \quad (\text{PVED})$$

where $E[\cdot]$ denotes the expectation operator. This model permits negative $d(t)$ that reflect capital contributions. $d(t)$ should in fact be referred to as dividends net of capital contribution but we will keep referring it to simply dividends for the sake of brevity.

The assumption of a constant discount factor will be made throughout this work. Non constant discount rates occur under a non-flat term structure of risk-free interest rates or when the rate of return required by the stockholders change in time (Feltham and Ohlson 1999). Taking this into account greatly complexifies the analysis, distract the attention away from our purpose of equity valuation onto utility functions (Rubinstein 1976) and will thus not be considered for the sake of clarity. In subsequent sections, the generic discount rate R may be replaced the riskless interest rate r under risk neutrality or the return on equity e when investors are risk-averse.

2.2 Residual income

Central to accounting-based valuation models, the clean surplus relation relates equity book value $bv(t)$ to net earnings $x(t)$ and dividends:

$$bv(t) = bv(t-1) + x(t) - d(t) \quad (\text{CSR})$$

This relation implies that all changes in book value are reported as either income or dividends. Although it may seem like an additional assumption, CSR can be verified by any firm as long as you define the proper earnings; in this relation, income should be seen as a plug variable rather than a pre-defined accounting concept. Thus CSR-based equity valuation methods do not favor any accounting system in particular and earnings are equally relevant to value in all accounting systems⁸.

We now define residual income $ax(t)$ as the difference between net income and a capital charge at the discount rate R :

⁸ See Dumontier (1998) for empirical evidence of this.

$$ax(t) = x(t) - R \cdot bv(t-1) \quad (\text{RI})$$

Residual income is very similar in nature to a project's NPV⁹ and Stewart's (1991) EVA¹⁰, i.e. they are a measure of whether the company is creating or destroying value, with the difference that EVA is written in terms of operating income and book capital while residual income is written in terms of total income and book value. The notation $ax(t)$ refers to Ohlson's naming of residual income, abnormal earnings. We however prefer to use residual income as it is more widely called by the financial community.

Combining PVED, CSR and RI leads to an alternate representation of the firm's equity, known today as residual income valuation:

$$\begin{aligned} P(t) &= \sum_{\tau=1}^{\infty} \frac{E[x(t+\tau) + bv(t+\tau-1) - bv(t+\tau)]}{(1+R)^\tau} \\ \Leftrightarrow P(t) &= \sum_{\tau=1}^{\infty} \frac{E[(1+R)bv(t+\tau-1) - bv(t+\tau) + ax(t+\tau)]}{(1+R)^\tau} \\ \Leftrightarrow P(t) &= \sum_{\tau=1}^{\infty} \frac{E[bv(t+\tau-1)]}{(1+R)^{\tau-1}} - \sum_{\tau=1}^{\infty} \frac{E[bv(t+\tau)]}{(1+R)^\tau} + \sum_{\tau=1}^{\infty} \frac{E[ax(t+\tau)]}{(1+R)^\tau} \\ \Leftrightarrow P(t) &= bv(t) + \sum_{\theta=1}^{\infty} \frac{E[bv(t+\theta)]}{(1+R)^\theta} - \sum_{\tau=1}^{\infty} \frac{E[bv(t+\tau)]}{(1+R)^\tau} + \sum_{\tau=1}^{\infty} \frac{E[ax(t+\tau)]}{(1+R)^\tau} \\ \Leftrightarrow P(t) &= bv(t) + \sum_{\tau=1}^{\infty} \frac{E[ax(t+\tau)]}{(1+R)^\tau} \quad (\text{RIV}) \end{aligned}$$

Although strictly equivalent to PVED, RIV has had the favors of academics because it shifts the focus from wealth distribution (dividends) to wealth creation (residual income). In that sense, equity valuation reconciles with the Modigliani-Miller (1961) theory of dividend irrelevancy through RIV. Residual income valuation also looks attractive to accountants as it reconnects (financial) equity valuation to their long-known concept of (accounting) goodwill, defined as the difference between the market and book values of a firm.

Directly from RIV, one can derive the following expression for the firm's goodwill $g(t)$:

$$g(t) = P(t) - bv(t) = \sum_{\tau=1}^{\infty} \frac{E[ax(t+\tau)]}{(1+R)^\tau}$$

known as early as Preinreich (1936).

9 Net Present Value

10 Economic Value Added

2.3 Linear information model

Ohlson's contribution lies in the additional specification of the time-series behavior of residual income. A simple linear information model formulates the dynamics of residual income and of information “other than”¹¹ residual income $v(t)$:

$$\begin{cases} ax(t+1) = \omega ax(t) + v(t) + \varepsilon_1(t) \\ v(t+1) = \gamma v(t) + \varepsilon_2(t) \end{cases} \quad (\text{LIM})$$

where the disturbance terms $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are two zero-mean random variable and where the parameter ω and γ are fixed and known, in the sense that the firm's economic environment and accounting principles determine ω and γ . We restrict ω and γ to be positive and less than 1 for stability.

From an operational point of view, historical values for ω and γ will be computed using classical regression techniques. As with the market beta, it could be interesting to use industry values instead of firm values as they are more stable and may be a better measure of the long-term values for ω and γ (Kaplan and Ruback 1995; Fama and French 1997). Alternatively, if one considers that markets are efficient and give the true price of the firm, OM can be solved for implied values of ω and γ .

Embedded in LIM is the assumption that $ax(t)$ and $v(t)$ are stationary processes; otherwise they should not be modeled using auto-regressive processes. A mean-reverting residual income process is coherent with economics where firms tend to lose any competitive advantage over time¹²; it is also consistent with the accounting principle of depreciating goodwill over time (Richard 1992).

Through $v(t)$ all non-accounting information makes its way into the valuation. More specifically, $v(t)$ can be re-written as:

$$v(t) = E[ax(t+1)] - \omega ax(t)$$

and thus be primarily interpreted as unpredicted growth.

Let's define the 2-by-2 matrix:

$$M = \frac{1}{(1+R)} \begin{pmatrix} \omega & 1 \\ 0 & \gamma \end{pmatrix}$$

11 Readers should not look for much meaning in this loose definition, due to Ohlson, and make early conclusions such as that $v(t)$ is uncorrelated to $ax(t)$, which is untrue. $v(t)$ is a catch-all variable that adds a degree of freedom in the specification of the information dynamics which allows for discrepancy between $P(t)$ and a price based on pure accounting data.

12 This is the dynamic Schumpeterian perspective on competition, in which firm characteristics are the determinants of economic rents. In contrast, industry structure is the determinant of economic rents in the static neoclassical perspective. See Mueller (1990).

LIM can be expressed as:

$$\begin{pmatrix} ax(t+1) \\ v(t+1) \end{pmatrix} = (1+R)\mathbf{M} \begin{pmatrix} ax(t) \\ v(t) \end{pmatrix} + \begin{pmatrix} \epsilon_1(t) \\ \epsilon_2(t) \end{pmatrix}$$

Under the expectation operator,

$$E \left[\begin{pmatrix} ax(t+1) \\ v(t+1) \end{pmatrix} \right] = (1+R)\mathbf{M} \begin{pmatrix} ax(t) \\ v(t) \end{pmatrix}$$

Recursively, we have:

$$E \left[\begin{pmatrix} ax(t+\tau) \\ v(t+\tau) \end{pmatrix} \right] = (1+R)^\tau \mathbf{M}^\tau \begin{pmatrix} ax(t) \\ v(t) \end{pmatrix}$$

Thus,

$$P(t) = bv(t) + (10) \sum_{\tau=1}^{\infty} \mathbf{M}^\tau \begin{pmatrix} ax(t) \\ v(t) \end{pmatrix}$$

The characteristic roots of the trigonal matrix \mathbf{M} are $\frac{\omega}{1+R}$ and $\frac{\gamma}{1+R}$. Because the maximum characteristic root is less than 1, the above \mathbf{M} -series converges and:

$$P(t) = bv(t) + (10) \mathbf{M} (\mathbf{1} - \mathbf{M})^{-1} \begin{pmatrix} ax(t) \\ v(t) \end{pmatrix}$$

where

$$(\mathbf{1} - \mathbf{M})^{-1} = \frac{1+R}{(1+R-\omega)(1+R-\gamma)} \begin{pmatrix} 1+R-\gamma & 1 \\ 0 & 1+R-\omega \end{pmatrix}$$

Finally, the Ohlson model for equity valuation writes:

$$P(t) = bv(t) + \frac{\omega}{(1+R-\omega)} ax(t) + \frac{(1+R)}{(1+R-\omega)(1+R-\gamma)} v(t) \quad (\text{OM})$$

We conclude that the firm's market value equals its book value adjusted for current profitability as measured by $ax(t)$ and for future profitability as measured by $v(t)$.

3 Formal connection with the Discounted Cash Flow method

In this section, we show that RIV and DCF are formally equivalent.

3.1 Discounted cash flows under risk neutrality

By definition,

$$bv(t) = oa(t) + fa(t)$$

where $fa(t)$ denotes the financial assets net of debt¹³ and $oa(t)$ the operating assets.

Each asset contributes to earnings:

$$x(t) = fx(t) + ox(t)$$

where $fx(t)$ denotes the financial income and $ox(t)$ the operating income, net of tax.

Under risk neutrality, the riskless interest rate r is the rate to be used throughout the firm. Then,

$$fx(t) = r \cdot fa(t-1)$$

At the end of the period, free cash flows $c(t)$ from operations (net of capital expenditures):

$$c(t) = ox(t) - oa(t) + oa(t-1)$$

are transferred to financial assets, leading to the following financial assets relation:

$$fa(t) = fa(t-1) + fx(t) + c(t) - d(t) = (1+r)fa(t-1) + c(t) - d(t) \quad (\text{FAR})$$

Finally, PVED and FAR lead to the well-known discounted cash flow formula:

$$\begin{aligned} P(t) &= \sum_{\tau=1}^{\infty} \frac{E[(1+r)fa(t+\tau-1) - fa(t+\tau) + c(t+\tau)]}{(1+r)^\tau} \\ \Leftrightarrow P(t) &= \sum_{\tau=1}^{\infty} \frac{E[fa(t+\tau-1)]}{(1+r)^{\tau-1}} - \sum_{\tau=1}^{\infty} \frac{E[fa(t+\tau)]}{(1+r)^\tau} + \sum_{\tau=1}^{\infty} \frac{E[c(t+\tau)]}{(1+r)^\tau} \\ \Leftrightarrow P(t) &= fa(t) + \sum_{\theta=1}^{\infty} \frac{E[fa(t+\theta)]}{(1+r)^\theta} - \sum_{\tau=1}^{\infty} \frac{E[fa(t+\tau)]}{(1+r)^\tau} + \sum_{\tau=1}^{\infty} \frac{E[c(t+\tau)]}{(1+r)^\tau} \\ \Leftrightarrow P(t) &= fa(t) + \sum_{\tau=1}^{\infty} \frac{E[c(t+\tau)]}{(1+r)^\tau} \quad (\text{DCF}) \end{aligned}$$

DCF is thus formally equivalent to PVED and RIV under risk neutrality.

¹³ $fa(t)$ can and most probably will be negative.

3.2 The weighted average cost of capital

Under risk, the discount factor in DCF must take into account the investor's risk aversion. We now show that this new discount rate R_c is indeed equal to the weighted average cost of capital $wacc$ used by the analyst in lieu of r in DCF above, when the cost of debt and financial leverage are assumed constant (Miles and Ezzell 1980).

At the end of the period, the cash flow to equity equals:

$$c(t+1) + T_x i_B B(t) + (B(t+1) - B(t)) - i_B B(t) + P(t+1)$$

where T_x is the firm's income tax rate, i_B the firm's cost of debt before tax and $B(t)$ the net debt $-fa(t)$. T_x and i_B are assumed constant throughout the life of the firm. The constituents of the cash flow to equity are the firm's free cash flows at the end of the period $c(t+1)$, plus the tax savings on interest payment $T_x i_B B(t)$, plus the change in financial liabilities $B(t+1) - B(t)$, plus the equity's future value $P(t+1)$. Note that the above cashflow minus the equity's future value can be referred to as the theoretical dividend and used in place of actual dividends to perform PVED valuation for firms that do not pay dividends (Batsch 2001).

Dividing by $P(t)$, we yield the definition of the return on equity e :

$$\frac{c(t+1) + T_x i_B B(t) + (B(t+1) - B(t)) - i_B B(t) + P(t+1)}{P(t)} = 1 + e \quad (\text{ROE})$$

The firm's total value is the sum of the market value of debt and equity:

$$V(t) = P(t) + B(t)$$

We now assume a constant financial leverage L :

$$\frac{B(t)}{V(t)} = L \Leftrightarrow B(t) = L V(t) \Leftrightarrow P(t) = (1 - L) V(t)$$

Introducing the financial leverage and firm's value in ROE¹⁴:

$$\begin{aligned} & \frac{c(t+1) + V(t+1)}{(1-L)V(t)} + \frac{(T_x i_B - i_B - 1)L}{1-L} = 1 + e \\ \Leftrightarrow & (1 + R_c) + (T_x i_B - i_B - 1)L = (1 + e)(1 - L) \\ \Leftrightarrow & R_c = (1 - L)e + L(1 - T_x)i_B = (1 - L)e + L i = wacc \end{aligned} \quad (\text{WACC})$$

where i is the firm's cost debt net of corporate taxes.

The proper discount rate to use for the DCF method under risk aversion is indeed the weighted average of the cost of equity and cost of debt net of corporate taxes. This result is valid under the assumption that both the cost of debt and financial leverage remain constant throughout the life of the firm.

¹⁴ Refer to the Appendix for intermediate calculations.

3.3 The rate of growth

Because cash flows cannot be forecasted over an infinite horizon, in practice analysts divide time in two periods, a finite period of explicit forecast of accounting data (years 1 to T) and a continuation period where an assumption of constant growth will be made to compute a terminal value:

$$\left| \begin{aligned} P(t) &= fa(t) + \sum_{\tau=1}^T \frac{E[c(t+\tau)]}{(1+wacc)^\tau} + \frac{TV_{dcf}(t+T)}{(1+wacc)^T} \\ P(t) &= bv(t) + \sum_{\tau=1}^T \frac{E[ax(t+\tau)]}{(1+e)^\tau} + \frac{TV_{riv}(t+T)}{(1+e)^T} \end{aligned} \right.$$

In the literature, the simple constant growth model is attributed to Gordon and Shapiro (1956) and results in the following terminal values:

$$\left| \begin{aligned} TV_{dcf}(t) &= \sum_{\tau=1}^{\infty} \frac{(1+g)^{\tau-1} E[c(t+1)]}{(1+wacc)^\tau} = \frac{E[c(t+1)]}{wacc-g} \\ TV_{riv}(t) &= \sum_{\tau=1}^{\infty} \frac{(1+g)^{\tau-1} E[ax(t+1)]}{(1+e)^\tau} = \frac{E[ax(t+1)]}{e-g} \end{aligned} \right.$$

where g is the growth rate of free cash flows and abnormal earnings.

There seems to be diverging points of view among researchers on whether the same growth rate should be used throughout the firm for the continuation period (Batsch 1999; Levin and Olsson 2000). We prove here that the same growth rate must in fact be used.

At any date t inside the continuing period, the valuation methods yield:

$$\left| \begin{aligned} P(t) &= \frac{E[d(t+1)]}{e-g_d} \\ P(t) &= fa(t) + \frac{E[c(t+1)]}{wacc-g_c} \\ P(t) &= bv(t) + \frac{E[ax(t+1)]}{e-g_{ax}} \end{aligned} \right.$$

where g_d , g_c , g_{ax} are the growth rates of dividends, free cash flows and income. We immediately infer that $P(t)$ grows at the constant rate g_d , because it is proportional to the dividend, and that $fa(t)$ grows at the constant rate g_d because of constant financial leverage. Then, $g_c = g_d = g$ because of FAR¹⁵. Let's now denote g_{bv} and g_x the growth rates for book value and income. Using Modigliani-Miller's dividend irrelevancy argument, we state that the steady-state modelization of the firm does not depend on the level of dividend distribution and that CSR holds for $d(t) = 0$. Hence the growth rates of dividends, earnings and book value are equal. Finally g_{ax} is equal to g through RI.

¹⁵ Refer to the Appendix for a detailed proof.

4 Implementing residual income valuation

The work of Ohlson received a warm welcome from the academic community (Bernard 1995) and was promptly followed by a series of empirical studies in an attempt to validate the model and compare its efficiency against the traditional method used by financial analysts based on free cash flows (Copeland et al. 1994). The very first studies did not however implement the linear information model and were merely testing the long-known residual income valuation model (Lo and Lys 2000). While we showed in the previous section that DCF and RIV are in fact formally equivalent, these studies did not find so and concluded on the superiority of RIV over discounting free cash flows.

In this section, we prove that the differences found in these studies come in fact from common mistakes in applying DCF and RIV and that these valuation methods should indeed yield the same numerical results, even under finite horizon, as long as the accounting data in the forecasting period and the steady state modelization of the firm in the continuation period are coherent. The empirical studies suffer from the following mistakes: inconsistent forecast error, incorrect discount rate, and missing cash flow, as labeled by Lundholm and O'Keefe (2001). We now review each of them in details.

4.1 Inconsistent forecast error

When modeling the firm in steady state starting at year T , primary accounting data, e.g. $x(t)$, are set to grow at the constant rate g . Cash flows ($d(t)$, $c(t)$, $ax(t)$) however, defined through a difference in primary accounting data, do not start until year $T+1$ to grow at the constant rate g . The inconsistent forecast error consists of starting constant growth at year T instead for the cash flows and thus assuming:

$$\begin{cases} E [c(t+T+1)] = (1+g)c(t+T) \\ E [ax(t+T+1)] = (1+g)ax(t+T) \end{cases}$$

in the calculation of the terminal values.

Although this error may at first look benign and not to introduce any bias between the valuation method, i.e. it is equivalent to shortening the forecasting period by one year, this is not the case as shown now in calculating the resulting forecast errors.

The forecast error in the RIV model is given by:

$$\begin{aligned} ax(t+T+1) - (1+g)ax(t+T) &= (x(t+T+1) - e bv(t+T)) - (1+g)(x(t+T) - e bv(t+T-1)) \\ &= -e(bv(t+T) - (1+g)bv(t+T-1)) \\ &= e(g - g_T)bv(t+T-1) \end{aligned}$$

where g_T is the growth in $bv(t)$ between $T-1$ and T .

The forecast error in the DCF model is given by:

$$\begin{aligned} c(t+T+1) - (1+g)c(t+T) &= (ox(t+T+1) - oa(t+T+1) + oa(t+T)) - (1+g)c(t+T) \\ &= oa(t+T) - (1+g)oa(t+T-1) \\ &= (g'_T - g)oa(t+T-1) \end{aligned}$$

where g'_T is the growth in $oa(t)$ between $T-1$ and T .

When $(g_T - g)$ and $(g'_T - g)$ are positive, which should be typically the case¹⁶, we see that the inconsistent forecasting error introduces valuation errors of opposite direction, and of different magnitude, in the RIV and DCF models.

This error is very common in the empirical literature. Penman and Sougiannis (1998) performed an ex-post¹⁷ comparison of RIV and DCF, using a 20-year sample (1973 to 1992¹⁸) of Compustat data. On page 355, Penman and Sougiannis say to be using the first 10 years when testing DCF and RIV over a finite horizon of 10 years. When setting $T=10$ in equation (10) on page 352, it turns out that they need the 1993 data to compute the terminal value, which is not included in the 20-year period. They must have then used the 1992 data to compute the 1993 data through steady-state growth and hence generated this error.

Francis et al. (2000) performed an ex-ante¹⁹ comparison of RIV and DCF, using a 5-year sample of Value Line data. Value Line reports analyst estimates of financial statements for up to 5 years²⁰. On page 53, they say to be using the RIV and DCF formula with a finite horizon of 5 years. They must then be needing forecasts for year 6 which is not provided by Value Line. Courteau et al. (2001) used the same source of data, value Line, and the same finite horizon statement of DCF and RIV with $T=5$; similarly they must have run into this inconsistent forecast error.

16 This is consistent with the life cycle of firm, whose growth rate starts on higher levels then lowers down to the inflation rate, or the growth rate in gross domestic product, in the long term.

17 Ex-post empirical evaluation of equity valuation models consists of going back in time and using realized earnings as a proxy for expected earnings. The benefit of this approach is that the data is readily available while it is only recently, and in a limited fashion, that analyst forecasts are collected and archived in databases. To reduce the bias introduced by the unpredictable component of realized data, value estimates are typically performed at the portfolio level where the unpredicted components average out.

18 The description of the sample period is not coherent in the paper, probably due to the long gestation of the experiment. We retain the 20-year period as it look the most consistent.

19 As opposed to ex-post empirical studies, ex-ante evaluations use true expected earnings (analyst forecasts) and are then able to work at the level of an individual firm.

20 More precisely, Value Line provides estimates for the current year (t), the following year ($t+1$) and the 3-to-5 years period. Francis et al. used this last data uniformly for years $t+3$, $t+4$, $t+5$ and computed the year $t+2$ data as the average of year $t+1$ and $t+3$ data.

4.2 Incorrect discount rate

The use of WACC for the DCF method is valid under the assumption that the financial leverage is constant. Generally, this does not hold true in the forecasted balance sheets used in empirical studies. Francis et al. (2000) exhibits this error as they say, on page 52, to assume *wacc* “constant across the forecast horizon”. In their study, *wacc* is computed on the target financial structure of the long-term continuation period; this value is known as *long-term wacc*.

One alternative is to the following valuation equation derived from PVED and FAR:

$$P(t) = fa(t) + \sum_{\tau=1}^{\infty} \frac{E[c(t+\tau) + fx(t+\tau) - R fa(t+\tau-1)]}{(1+R)^\tau}$$

which is not requiring the use of WACC thus not constraining the increment in financial assets. This is done in Courteau et al. (2001).

Another alternative would be to use the APV²¹ method (Myers 1974) when future debt of the firm is deterministic. In this method, the value of the (levered) firm equals the value of the “otherwise equivalent”²² unlevered firm plus the value of the tax shield from debt financing. The valuation formula writes:

$$P(t) = \sum_{\tau=1}^{\infty} \frac{E[c(t+\tau)]}{(1+e_0)^\tau} + \sum_{\tau=1}^{\infty} \frac{iT_x B(t+\tau-1)}{(1+i)^\tau}$$

where e_0 is the cost of equity of the equivalent unlevered firm. APV has been the approach of choice when dealing with LBO's²³ and highly-leveraged transactions²⁴ where leverage ratios are highly random.

4.3 Missing cash flow

This error arises for instance when the forecasted accounting data violates CSR. As noted in Courteau et al. (2001), it occurred 12 percent of the time in the Value Line forecasts and the error is “within $\pm 5\%$ of book values”. Since the authors “do not force the CSR to hold when forecasted violations of CSR exist” as written on page 645, the equivalence of the valuation models is no longer guaranteed to hold. Similarly, Francis et al. (2000) did not adjust for dirty surplus and exhibits this error.

To size the impact on dirty surplus accounting, let's rewrite RIV and OM in terms of an alternate measure of income $y(t)$ and its corresponding dirty surplus income $z(t) = x(t) - y(t)$ (Lo & Lys

21 Adjusted Present Value

22 In the sense of Modigliani and Miller (1958).

23 Leveraged Buy Out

24 See Kaplan and Ruback (1995) for such an implementation of APV.

2000). Similarly to RI, we define a residual income $ay(t)$ based on $y(t)$ as follows:

$$ay(t) = y(t) - Rbv(t-1)$$

and abnormal clean surplus earnings then write:

$$ax(t) = x(t) - Rbv(t-1) = y(t) + z(t) - Rbv(t-1) = ay(t) + z(t)$$

Under dirty surplus accounting, RIV becomes:

$$P(t) = bv(t) + \sum_{\tau=1}^{\infty} \frac{E[ax(t+\tau)]}{(1+R)^\tau} = bv(t) + \sum_{\tau=1}^{\infty} \frac{E[ay(t+\tau)]}{(1+R)^\tau} + \sum_{\tau=1}^{\infty} \frac{E[z(t+\tau)]}{(1+R)^\tau}$$

Researchers typically use alternate measure of income instead of the clean surplus measure of income $x(t)$. One reason for it is that they tend to split income between income before extraordinary items (that would be $y(t)$) and extraordinary items ($z(t)$) which are considered more transitory and should be assigned a different time-series behavior.

Let's assume $z(t)$ is not affected by non-accounting information $v(t)$. Under dirty surplus accounting, LIM thus becomes:

$$\begin{cases} ay(t+1) = \omega ay(t) + v(t) + \varepsilon_1(t) \\ v(t+1) = \gamma v(t) + \varepsilon_2(t) \\ z(t+1) = \theta z(t) + \varepsilon_3(t) \end{cases}$$

where $\varepsilon_3(t)$ is an additional disturbance term and θ a positive parameter smaller than 1.

Under dirty surplus accounting, the OM valuation formula finally becomes:

$$P(t) = bv(t) + \frac{\omega}{(1+R-\omega)} ay(t) + \frac{(1+R)}{(1+R-\omega)(1+R-\gamma)} v(t) + \frac{\theta}{R-\theta} z(t)$$

We see that missing dirty surplus enters the valuation equation additively. Not adjusting for dirty surplus will bias the valuation of $P(t)$ upward or downward depending on the sign of $(R-\theta)$.

Another form of the missing cash flow error occurs when Value Line forecasts non-operating income. While it will be included in RIV, as $ax(t)$ is computed based on total income $x(t)$, it may not be included in the DCF value. Indeed, Francis et al. define operating income as:

$$ox(t) = (\text{sales} - \text{operating expenses} - \text{depreciation expense}) (1 - \text{tax rate})$$

using the corresponding fields in Value Line and estimate $fa(t)$ directly as the book value of debt. Thus the non-operating income, that may exist in $x(t)$, will never be valued in DCF.

To conclude on these three empirical mistakes, we report on the sample correction calculated by Lundholm and O'Keefe (2001) to Francis et al. (2000) in order to recover the difference in firm's value produced by RIV and DCF. On average, DCF exceeded RIV by \$5.86 per share in Francis et al. (2000), for an average stock price of \$31.27. Computing the forecast error using the equation

given in this section, one finds that it amounts to \$6.75. Thus, by correcting only for the inconsistent forecast error, one recovers most of the discrepancy and the difference in valuation is down to \$0.86, i.e. less than 3% of the average stock price.

5 Implementing linear information models

The previous section concluded that it was pointless to empirically compare OM and DCF if LIM was not implemented. In this section, we show that in fact, because of the linear nature of LIM, there is nothing in its current formulation that makes it superior to the traditional constant perpetuity model (Lo and Lys 2000). We then report on a second set of empirical studies that did implement the linear information model. Their findings are consistent with our conclusion. Finally, we conclude this section on what we believe is the main contribution of the LIM; because it provides a formal framework, it is the place where business knowledge on income determinants and academic research on valuation modelization meet to refine the time-series behavior of income.

5.1 Gordon-Shapiro as a linear information model

The original Gordon and Shapiro (1956) model was stated in terms of dividends as follows:

$$\begin{cases} d(t) = T_d x(t) \\ x(t) = \rho b v(t-1) \end{cases} \quad (\text{GS})$$

where T_d is the dividend distribution rate and ρ the book return on equity.

At first look, this dividend model does not relate to the Ohlson (1995) linear information model. Indeed Ohlson (1995) departed from a dividend-based model to avoid the contradiction of dividend irrelevancy. However we now show that, under CSR, it can be rewritten as a special case of the LIM.

First we verify that the above model lead to accounting data growing at a constant rate g as applied in the previous section.

$$\begin{cases} b v(t+1) = b v(t) + x(t+1) - d(t+1) = b v(t) + (1 - T_d) x(t+1) = (1 + g) b v(t) \\ x(t+1) = \rho b v(t) = (1 + g) x(t) \\ d(t+1) = T_d x(t+1) = (1 + g) d(t) \end{cases}$$

where $g = \rho (1 - T_d)$.

In addition, we can write RI as:

$$a x(t+1) = x(t+1) - e b v(t) = (\rho - e) b v(t) = (1 + g) a x(t)$$

which is in essence LIM with $v(t) = 0$.

If we feed this model in lieu of LIM in RIV, we get the following valuation equation:

$$V(t) = b v(t) + \frac{1 + g}{e - g} a x(t)$$

which is in fact a OM model with parameters:

$$\begin{cases} \omega = 1 + g \\ \gamma = 0 \end{cases}$$

Moreover, while OM adds the information on unpredicted growth through $v(t)$, DCF does it through a finite forecasting period. Both models then encompass the same set of information and should again yield similar results. We thus conclude that the LIM did not introduce anything that would make OM superior to DCF.

5.2 Empirical studies

Consistent with our conclusion above, the empirical literature could not find any superiority of OM compared to DCF. In 1999, Myers tested the OM and treated the “other information” in two ways. First, it ignored $v(t)$ on the basis that it is unobservable; this is his LIM1 model. Second, it proxied $v(t)$ using order backlog; this is his LIM4 model. The choice of order backlog is mainly practical; while many variables²⁵ may proxy the sense of growth in $v(t)$, order backlog is one of the few variables readily available. Myers also tested derivative models of the OM that account for accounting conservatism²⁶. Myers concluded that the OM largely understated the firm value and that the LIM does not capture the time-series behavior of accounting data.

Again in 1999, Dechow et al. implemented the OM using the forecast $ex(t)$ of period $t+1$ earnings as a proxy for other information. $v(t)$ can then be expressed as:

$$v(t) = E[ax(t+1)] - \omega ax(t) = ex(t) - Rbv(t) - \omega ax(t)$$

Substituting in OM, we have:

$$P(t) = \alpha_1 bv(t) + \alpha_2 ax(t) + \alpha_3 ex(t)$$

where

$$\begin{cases} \alpha_1 = \frac{(1+R-\omega)(1+R-\gamma) - R(1+R)}{(1+R-\omega)(1+R-\gamma)} \\ \alpha_2 = -\frac{\omega\gamma}{(1+R-\omega)(1+R-\gamma)} \\ \alpha_3 = \frac{1+R}{(1+R-\omega)(1+R-\gamma)} \end{cases}$$

Dechow et al. (1999) concluded that their implementation of the OM “provides only minor improvements over existing attempts to implement the dividend-discounted model by capitalizing short-term earnings’ forecasts in perpetuity.” Unlike Myers (1999) though, their results “generally support Ohlson’s information dynamics.”

²⁵ Myers (1999) mentioned “new patents, regulatory approval of a new drug for pharmaceutical companies, new long-lived contracts and order backlog.”

²⁶ See Section 4.3.1.

5.3 Extending the LIM

Although LIM did not introduce any information that made OM more relevant in equity valuation, the formalization of LIM has set the ground for the incorporation of such information. We believe that this is where the added value of the Ohlson model lies; it has given a formal framework in which additional accounting and non-accounting information can be brought in to explain the price of a firm's equity. It brings a methodology to a field which has been synonymous to ad hoc regression tests (from academics) and hand waving (from business analysts). However to surpass today's methods based on forecasting free cash flows, the information dynamics will need to first integrate the information already contained in the forecasted cash flows then find additional and non-redundant information. A complementary line of research would be to better fit the time-series behavior of accounting data than the simple linear dynamics in as DCF and OM. We now describe in more depth three sample extensions of the LIM.

5.3.1 Accounting conservatism

Feltham and Ohlson (1995) extended LIM to account for accounting conservatism:

$$\begin{cases} ax(t+1) = \omega_{11} ax(t) + \omega_{12} oa(t) + v_1(t) + \varepsilon_1(t) \\ oa(t+1) = \omega_{22} oa(t) + v_2(t) + \varepsilon_2(t) \\ v_1(t+1) = \gamma_1 v_1(t) + \varepsilon_3(t) \\ v_2(t+1) = \gamma_2 v_2(t) + \varepsilon_4(t) \end{cases}$$

where the disturbance terms $\varepsilon_1(t)$, $\varepsilon_2(t)$, $\varepsilon_3(t)$ and $\varepsilon_4(t)$ are zero-mean random variables and where the parameter ω_{ii} and γ_i are restricted for stationarity as follows:

$$\begin{cases} 0 \leq \omega_{11} < 1 \\ 1 \leq \omega_{22} < 1 + R \\ \omega_{12} \geq 0 \\ |\gamma_i| < 1 \end{cases}$$

ω_{11} is related to persistence in residual income, ω_{22} to long-term growth in operating assets and ω_{12} to accounting conservatism, defined as follows; accounting is unbiased (resp. conservative) if $E[g(t)] \rightarrow 0$ (resp. > 0) as $t \rightarrow \infty$. Above if $\omega_{12} = 0$ (resp. $\omega_{12} > 0$), accounting is unbiased (resp. conservative).

This new information dynamics leads to the following Feltham-Ohlson valuation model:

$$P(t) = bv(t) + \alpha_1 ax(t) + \alpha_2 oa(t) + \beta_1 v_1(t) + \beta_2 v_2(t)$$

where

$$\begin{cases} \alpha_1 = \frac{\omega_{11}}{(1+R-\omega_{11})} \\ \alpha_2 = \frac{\omega_{12}(1+R)}{(1+R-\omega_{22})(1+R-\omega_{11})} \\ \beta_1 = \frac{1+R}{(1+R-\omega_{11})(1+R-\gamma_1)} \\ \beta_2 = \frac{\alpha_2}{1+R-\gamma_2} \end{cases}$$

In the Feltham and Ohlson (1995) model, the firm's market value thus equals its book value adjusted for current profitability as measured by $ax(t)$, for accounting conservatism through a second occurrence²⁷ of $oa(t)$, and for other information $v_1(t)$ and $v_2(t)$.

Accounting conservatism was tested in Myers (1999) in two ways. He first implemented the Feltham and Ohlson (1995) model; this was Myers' LIM2 model. Arguing that the “actual effects of conservatism may be more complex” and in fact happen in both the income and book value, he implemented a second model where book value and earnings are functions of cash receipts $cr(t)$ and capital investment in assets $capx(t)$, and explicated a linear time-series dynamics for $cr(t)$ and $capx(t)$; this is his LIM3 model. Both models failed however to “accurately characterized the time-series of residual income” and Myers concluded that better models were needed.

5.3.2 Accounting for intangibles

While today's expenditures on intangibles, such as R&D and advertising, ensures the future profitability of a firm, its full and immediate expensing, as prescribed by FASB²⁸, reduces the firm's assets and bias downward its valuation in traditional methods. Research has been very active in demonstrating that investments in RD and advertising are positively related to the firm's value²⁹.

As in Sougiannis (1994), the Ohlson model can be used to model the impact of R&D expenditures on the firm's value and to estimate the investment value of R&D. Let $v(t)$ denote here the firm's R&D expenditures³⁰.

Net accounting earnings may be written as:

$$x(t) = (x_B(t) - v(t)) \cdot (1 - T_x)$$

where T_x is the firm's income tax rate and $x_B(t)$ earnings before expensing R&D at time t .

²⁷ $oa(t)$ is already present in $bv(t)$.

²⁸ Financial Accounting Standards Board

²⁹ See Cañibano et al. (1999) for an extensive literature review.

³⁰ This is a simplified version of the work presented in Sougiannis (1994) as we do not take into account the value relevance of lagged R&D expenditures. Specifically, in Sougiannis (1994), $v(t)$ is set to be the vector of R&D expenditures over the past N period.

Inserting the above equation into OM leads to:

$$P(t) = bv(t) + \beta_1 ax_B(t) + \beta_2 T_x v(t) + \beta_3 v(t)$$

where the β_i are functions of the LIM parameters and where

$$ax_B(t) = x_B(t)(1 - T_x) - Rbv(t-1)$$

represents a measure of residual income based on earnings before R&D expenditures.

In this model, the firm's market value thus equals its book value adjusted for current profitability as measured by $ax_B(t)$, for the tax shield $T_x v(t)$ resulting from the immediate expensing of R&D expenditures, and for future profitability as measured by today's R&D expenditures $v(t)$.

When empirically testing this model on a eleven-year period (1975 to 1985), Sougiannis found that a one-dollar R&D investment resulted in a two-dollar profit over a seven-year period, that an incremental one-dollar R&D investment increased market value by five dollars, and that the tax shield from R&D expensing was valued as earnings.

5.3.3 Time-series behavior

Orthogonal to the addition of complementary information into the information model, a more sophisticated time-series modelization can improve the effectiveness of the model by better fitting the evolution of the cash flows. A first idea is to consider that the persistence in residual income has an impact over a multi-year period.

We imagine that the number of years in the period would match the typical time-to-market of the industry the firms evolves in. In the high-end Unix computer industry for instance, new non-backward-compatible generations of products come out every four years or so. At every business cycle, which is pretty well synchronized across hardware vendors, the vendor that takes the technology lead is guarantee of an excess level of revenue for the upcoming four years through hardware upgrade, maintenance contracts and consulting fees.

Technically, adding delays in the persistence of residual income means modeling it using an AR(p) auto-regressive process. We now show that the original property of the model, that market value is equal to book value adjusted for current and future profitability, is preserved.

Let's define the p-by-p matrix:

$$M = \frac{1}{(1+R)} \begin{pmatrix} \omega_1 & \omega_2 & \omega_3 & \cdots & \omega_p \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}$$

LIM can be expressed as:

$$\begin{pmatrix} ax(t+p+1) \\ \vdots \\ ax(t+2) \end{pmatrix} = (1+R)\mathbf{M} \begin{pmatrix} ax(t+p) \\ \vdots \\ ax(t+1) \end{pmatrix}$$

Recursively, we have:

$$\begin{pmatrix} ax(t+p+\tau) \\ \vdots \\ ax(t+\tau) \end{pmatrix} = (1+R)^\tau \mathbf{M}^\tau \begin{pmatrix} ax(t+p) \\ \vdots \\ ax(t+1) \end{pmatrix}$$

Thus,

$$P(t) = bv(t) + \sum_{\tau=1}^p \frac{E[ax(t+\tau)]}{(1+R)^\tau} + \frac{1}{(1+R)^p} (1 \cdots 0) \sum_{\tau=1}^{\infty} \mathbf{M}^\tau \begin{pmatrix} ax(t+p) \\ \vdots \\ ax(t+1) \end{pmatrix}$$

The characteristic polynomial of \mathbf{M} is:

$$\lambda^p - \sum_{i=1}^p \frac{\omega_i}{(1+R)^i} \lambda^{p-i}$$

Under the assumption that the maximum characteristic root is less than 1, the above \mathbf{M} -series converges and:

$$P(t) = bv(t) + \sum_{\tau=1}^p \frac{E[ax(t+\tau)]}{(1+R)^\tau} + \frac{1}{(1+R)^p} (1 \cdots 0) \mathbf{M} (\mathbf{I} - \mathbf{M})^{-1} \begin{pmatrix} ax(t+p) \\ \vdots \\ ax(t+1) \end{pmatrix}$$

Finally,

$$P(t) = bv(t) + \sum_{i=1}^p \alpha_i E[ax(t+i)]$$

where the parameters α_i are function of the ω_i and R .

A second idea is to consider that residual income will be stationary by periods in the continuation period, due to changes in growth rates, accounting procedures and production technologies (Myers 1999). In fact, some financial firms already consider changes in growth rates but take a manual approach based on slicing the continuation period in the DCF method. At Associés en Finance³¹ for instance, equity valuation is carried out using a 4-period modelization (Hamon 2001): a 5-year forecast period, a pre-maturity period³², a maturity period³³ and a residual continuation³⁴ starting at

31 Go to <http://www.associes-finance.com/> for more information about Associés en Finance.

32 In the pre-maturity period, a normalized earnings-per-share is forecasted, the firm's price growth rate is assumed constant, the volume growth rate decays down to its maturity value, and the dividend distribution rate grows up to its maturity value.

33 In the maturity period, the firm's growth rate is aligned with the average growth rate of its sector.

34 In the continuation period, the firm's growth rate is aligned with the country's inflation rate; in the long run, the firm has lost its competitive edge and is no longer creating value.

year $N+26$. A formalization of this could take the form of an exponential auto-regressive (EXPAR) process (Haggan and Ozaki 1981) where residual income would be a non-linear function of its past values. A first-order EXPAR modelization for $ax(t)$ writes (Mignon 2001):

$$ax(t+1) = \left(\phi + \sum_{k=0}^s \beta_k ax(t)^k \exp(-\alpha ax(t)^2) \right) ax(t) + \varepsilon(t)$$

where $\varepsilon(t)$ is a white noise. We will not attempt here to develop this alternative further.

6 Conclusion

The Ohlson model extends the residual income model for equity valuation by specifying a linear model for the time-series behavior of residual income. As a result, stock price is related through a simple linear formula to current book value, current profitability and future profitability. Despite their common theoretical ground with the widely used discounted cash flow method, early empirical studies were conducted to compare the efficiency of the accounting-based OM and RIV models and of the DCF, on the perception that the OM and RIV bring the future forward in time. Indeed, while profits from a given investment appear immediately in accounting income, it will not appear in cash flows until the initial investment cost is offset by accumulated profits. As a consequence, the error-prone terminal value involved in equity valuation over a finite horizon is of less weight in accounting-based models than in DCF. This reasoning drove a series of empirical studies upon the publication of Ohlson (1995) which all concluded in the empirical superiority of accounting-based models.

In this work, we demonstrate that this claim is unfounded. First, we demonstrate the strict equivalence of RIV and DCF under the assumptions of constant cost of debt and constant financial leverage throughout the life of the firm. When implemented over a finite horizon, RIV and DCF still yield the same results as long as the steady-state modelizations of the firm in the continuation period under both methods are coherent, which the reviewed empirical studies failed to do. We note three common mistakes: the inconsistent forecast error, which results in starting the continuation period on two different years for each method; the incorrect discount error, which maintains the use of *wacc* while violating the assumption of constant financial leverage; and missing cash flow errors, due to dirty surplus for instance. Such errors underline the difficulty of consistent forecasts among different valuation paradigms. If one compensates for these errors on the data published in the empirical studies, RIV and DCF are shown to yield very similar results. Conclusions on the superiority of RIV over DCF were thus unfounded. Secondly, because of its linear nature, LIM is not conceptually superior to the traditional Gordon-Shapiro model used in DCF. In fact, Gordon-Shapiro can be rewritten as a linear information model as shown in this paper. As a consequence, OM is not more efficient nor accurate than DCF.

Despite our conclusion that the OM does not perform better than DCF, we still believe that it is a step forward in equity valuation. Through LIM, OM provides a framework to better capture the value drivers which should eventually yield better results in derivative OM models. With OM, the focus has shifted from algebra in pure financial approaches to business knowledge. In that sense

OM reconnects equity valuation with business sense. Studies that seek explanatory variables to business performance and value (Parienté 2000) will benefit from the formalization of the LIM, leading to reveal the driving forces behind the regression coefficients in the value equation, thus allowing more accurate forecast. In fact, we believe that, thanks to the OM, research can now focus on its single most important issue of identifying and forecasting earning drivers. As Myers (1999) writes it, “the main challenge in valuation is forecasting future earnings. The future research efforts of empiricists will be more useful if they are focused on this main issue (...)”. As a follow-on to this work, we suggest that a specific market is selected, value drivers are identified using econometrical techniques, a linear information model is established, a valuation equation is derived and an empirical study is led to measure the forecasting ability of the equation and the added value of the OM framework.

7 Appendix

7.1 From ROE to WACC

In WACC, we use the fact that $c(t+1) + V(t+1) = (1+R) V(t)$. Indeed:

$$\begin{aligned} V(t) &= \sum_{\tau=1}^{\infty} \frac{E[c(t+\tau)]}{(1+R)^\tau} \\ \Leftrightarrow V(t) &= c \frac{(t+1)}{1+R} + \frac{1}{1+R} \sum_{\tau=1}^{\infty} \frac{E[c(t+1+\tau)]}{(1+R)^\tau} \\ \Leftrightarrow V(t) &= c \frac{(t+1)}{1+R} + \frac{1}{1+R} V(t+1) \\ \Leftrightarrow (1+R)V(t) &= c(t+1) + V(t+1) \end{aligned}$$

It is tempting to write:

$$\begin{aligned} c(t+1) &= R V(t) \\ \Leftrightarrow (1+R)V(t) &= c(t+1) + V(t) \end{aligned}$$

but this is only true under the Gordon-Shapiro assumptions and a rate of growth for the cashflow equal to zero. In that case, $V(t)=V(t+1)$ and we indeed match the above generic case.

7.2 Relation between the rates of growth for cashflows and dividends

DCF yields the following relationship between the firm's value $V(t)$ and the cashflow $c(t+1)$:

$$\begin{aligned} V(t) &= fa(t) + \frac{c(t+1)}{w - g_c} \\ \Leftrightarrow V(t) &= \frac{L}{L-1} V(t) + \frac{c(t+1)}{wacc - g_c} \\ \Leftrightarrow \frac{wacc - g_c}{1-L} V(t) &= c(t+1) \end{aligned}$$

PVED and FAR yield this second relationship:

$$\begin{aligned} V(t) &= \frac{d(t+1)}{e - g_d} \\ \Leftrightarrow (e - g_d)V(t) &= c(t+1) - fa(t+1) + (1+i)fa(t) \\ \Leftrightarrow (e - g_d)V(t) &= c(t+1) + (i - g_d)fa(t) \\ \Leftrightarrow (e - g_d + \frac{L}{1-L}(i - g_d))V(t) &= c(t+1) + (i - g_d)fa(t) \\ \Leftrightarrow (e - g_d + \frac{L}{1-L}(i - g_d))V(t) &= c(t+1) \\ \Leftrightarrow \frac{wacc - g_d}{1-L} V(t) &= c(t+1) \end{aligned}$$

We now set equal the above two expressions for the cashflow:

$$\frac{wacc - g_c}{1 - L} V(t) = \frac{wacc - g_d}{1 - L} V(t)$$
$$\Leftrightarrow wacc - g_c = wacc - g_d$$
$$\Leftrightarrow g_c = g_d$$

and find that the rates of growth for the cashflow and the dividend are equal.

8 References

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